Need for Speed?
Exchange Latency and Liquidity

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Outline

Motivation

Model

Conclusion
NYSE's Fast-Trade Hub Rises Up in New Jersey
LSE goes live with faster trading system
CitiFX launches Velocity 2.0; stakes claim as the fastest platform in market

by Hamish Risk, Laurence Twelvetrees
February 8, 2010

NASDAQ OMX Launches INET Trading System
Question:
Question:
Is an even faster exchange good for liquidity?
Literature on HFT and adverse selection

1. *Theory:*
   HFTs fast/informed speculators...
   (Foucault, Hombert, and Roșu, 2015; Biais, Foucault, and Moinas, 2015)

2. *Evidence:*
   HFTs adverse select and get adverse selected.
   (Hendershott and Riordan, 2011; Baron, Brogaard, and Kirilenko, 2014; Brogaard, Hendershott, and Riordan, 2014)

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   ...or are on both sides.
   (Budish, Cramton, and Shim, 2015; Haas and Zoican, 2016)
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Topology of modern exchanges

Network-to-gateway latency

Gateway processing latency

Gateway-to-matching-engine latency

Exchange latency

Trader A
Trader B
Trader C
Trader D
Gateway 1
Gateway 2
Matcher
Matching engine
Our attempt on exchange speed and liquidity (in pictures)
Takeaways

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3. At the same time, a faster exchange allows the high-frequency market maker to update his quotes more quickly and reduce his payoff risk.
Takeaways

1. Lowering exchange latency can reduce liquidity.
2. A higher exchange speed makes a high-frequency market-maker duel with high-frequency bandits more often.
3. At the same time, a faster exchange allows the high-frequency market maker to update his quotes more quickly and reduce his payoff risk.
4. The net effect on liquidity depends on news-to-liquidity-trader ratio and HFT risk aversion.
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   HFTs are risk-averse with piece-wise linear utility (Moinas and Pouget, 2013):

   $$U^{HFT}(x) = \gamma x 1_{x<0} + x 1_{x\geq0},$$  \hspace{1cm} (1)
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HFTs choose between two strategies:

1.1 High-frequency market maker (HFM)

1.2 High-frequency bandit (HFB)

2. Uninformed and slow: Liquidity traders (LT).

Exchange

1. Limit order book.

2. Latency: HFTs send messages at \( t \), processed at \( t + \delta \).
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Asset

1. News arrives with probability $\alpha \tau$ in a period of length $\tau$. 

Common value jumps by $\pm \sigma$. 

LT arrival with probability $\mu \tau$ in a period of length $\tau$. 

Private value is $\pm \sigma'$, $\sigma' > \sigma$. 

Either zero or one event possible in a latency interval $\delta$. The probability of two or more events is ignored.

Common value $v_t$ can change in an interval $\delta$:

\[
\begin{cases} 
    v_t - \sigma & \text{("bad" news arrival)} \\
    v_t & \text{(no news arrival)} \\
    v_t + \sigma & \text{("good" news arrival)}
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Timing

Timing of the model is as follows:

- \( t = -1 \)
- \( t = 0 \)
- \( t = \delta \)

- HFM initial quotes
- Trigger event (news or LT arrival at matcher)
  - If news, HFM sends quote-cancel order and HFBs send quote-snipe order
- Latency \( \delta \)
- No further arrival, or LT order arrival at matcher, or news arrival (mutually exclusive events)

HFT orders’ arrival at matcher
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\[ \text{Latency } \delta \]

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1. At $t \in \{-1, 0\}$, HFTs decide whether to submit a market order, cancel limit orders, or both.
2. HFTs arrive at the market in random order. Market orders and cancellations execute, new price quotes are submitted.
Solution

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2. All HFTs take same action at $t = -1$.
   All HFM s and all HFBs take same action at $t = 0$. 

Two equilibrium types

There exists a risk-aversion threshold $\bar{\gamma}$ such that:

1. For $\gamma \leq \bar{\gamma}$, a sure-sniping equilibrium emerges.
2. For $\gamma > \bar{\gamma}$, a mixed-sniping equilibrium emerges.

Sniping equilibrium (baseline)

1. In equilibrium, HFTs are indifferent between HFM and HFB strategies.
2. Equilibrium half-spread $s^*$ nailed by indifference condition:
   $U_{HFM}(s^*) = U_{HFB}(s^*)$. 

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Let $U_{\text{HFB}}(s|\text{trade})$ be the expected HFB profit conditional on a trade ($s$ is half-spread, referred to as spread throughout):
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$$U_{\text{HFB}}(s|\text{trade}) = \left(1 - \frac{\mu \delta}{2} - \alpha \delta\right)(\sigma - s) + \alpha \delta \left[\frac{1}{2} (2\sigma - s) - \frac{1}{2}s\gamma\right]$$

- No event during latency delay
- News arrives during latency delay
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Each HFT is first to the market with probability $\frac{1}{H}$.

The HFB expected profit is:

$$U_{HFB}(s) = \frac{\alpha}{\mu + \alpha} \underbrace{\frac{1}{H}}_{\text{HFB first}} \underbrace{\overbrace{U_{HFB}(s|\text{trade}).}^\text{News before LT}}_{\text{No event during latency delay}}}$$
The HFM expected profit, $U_{HFB}(s)$, is:
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$$\frac{\mu}{\mu + \alpha} \begin{cases} \left(1 - \alpha \delta - \frac{\mu \delta}{2}\right) s + \frac{\mu \delta}{2} 2s + \alpha \delta \left(\frac{1}{2} (s + \sigma) + \frac{1}{2} (s - \sigma \gamma)\right) \\
\text{LT before news} \quad \text{No event/LT on same side} \quad \text{LT on opposite side} \quad \text{News event}
\end{cases}$$
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+ \frac{\alpha}{\mu + \alpha} \left( \frac{1}{2} \mu \delta (s - \sigma) \gamma + \frac{1}{2} \mu \delta (s + \sigma) \right)
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- LT before news
  - No event/LT on same side
  - LT on opposite side
  - News event
- News before LT
  - LT on news side
  - LT on no-news side
The HFM expected profit, \( U_{HFB}(s) \), is:

\[
\frac{\mu}{\mu + \alpha} \left[ \left( 1 - \alpha \delta - \frac{\mu \delta}{2} \right) s + \frac{\mu \delta}{2} \right] + \alpha \delta \left( \frac{1}{2} (s + \sigma) + \frac{1}{2} (s - \sigma) \gamma \right)
\]

\[
+ \frac{\alpha}{\mu + \alpha} \left( \frac{1}{2} \mu \delta (s - \sigma) \gamma + \frac{1}{2} \mu \delta (s + \sigma) \right)
\]

\[
+ \frac{\alpha}{\mu + \alpha} \frac{H - 1}{H} \left\{ \alpha \delta \left[ \frac{1}{2} (s - 2\sigma) \gamma + \frac{1}{2} s \right] + \left( 1 - \frac{\mu \delta}{2} - \alpha \delta \right) (s - \sigma) \gamma \right\}
\]

LT before news \( \Rightarrow \) No event/LT on same side \n\[
\left( 1 - \alpha \delta - \frac{\mu \delta}{2} \right) s + \frac{\mu \delta}{2}
\]

LT on opposite side \n\[
\alpha \delta \left( \frac{1}{2} (s + \sigma) + \frac{1}{2} (s - \sigma) \gamma \right)
\]

News event \n\[
\frac{1}{2} \mu \delta (s - \sigma) \gamma + \frac{1}{2} \mu \delta (s + \sigma)
\]

News before LT \( \Rightarrow \) LT on news side \n\[
\frac{1}{2} \mu \delta (s - \sigma) \gamma + \frac{1}{2} \mu \delta (s + \sigma)
\]

LT on no-news side \n\[
\alpha \delta \left[ \frac{1}{2} (s - 2\sigma) \gamma + \frac{1}{2} s \right] + \left( 1 - \frac{\mu \delta}{2} - \alpha \delta \right) (s - \sigma) \gamma
\]

News arrives during latency delay \n\[
\alpha \delta \left[ \frac{1}{2} (s - 2\sigma) \gamma + \frac{1}{2} s \right] + \left( 1 - \frac{\mu \delta}{2} - \alpha \delta \right) (s - \sigma) \gamma
\]

No event during latency delay
Sure-sniping equilibrium

Proposition 1

The following strategies for HFM and HFB constitute a unique equilibrium for \( \gamma < \bar{\gamma} \).

1. At \( t = -1 \), all HFTs submit one buy limit order at \( v_0 - s^* \) and one sell limit order at \( v_0 + s^* \). The first arriving HFT (picked randomly) fills the order book; we refer to this HFT as the HFM and to the other HFTs as HFBs.

2. A trigger event occurs at time \( t = 0 \). If the trigger event is a news arrival (i.e., if \( v_0 \neq v_{-1} \)), then the HFM submits a quote-cancel order and, at the same time, all HFBs submit a market order aimed at the stale quote on the news side of the book (i.e., the ask side if news was good or the bid side when news was bad).
Sure-sniping equilibrium
The equilibrium spread is

\[ s^* = \sigma \frac{\alpha [\delta \mu (2 + H) - 2\gamma (H + \delta \mu - 1) - 2]}{\alpha^2 \delta (\gamma - 1) (H - 2) - \mu H (2 + \delta \mu) - \alpha [2 + \delta \mu (H - 2) + 2\gamma (H - 1 + \delta \mu)]} \]
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1. Comparative statics: \( s^* \uparrow \alpha, s^* \uparrow \sigma, s^* \downarrow \mu, s^* \uparrow \gamma. \)
**Equilibrium spread**

The equilibrium spread is

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\]

1. Comparative statics: \( s^* \uparrow \alpha, s^* \uparrow \sigma, s^* \downarrow \mu, s^* \uparrow \gamma \).

2. Also, equilibrium spread \( s^* \uparrow H \).

   More HFBs lead to higher adverse selection costs for the (unique) HFM.
Equilibrium spread and exchange speed

Proposition 2
There exists $T_{\gamma,H}$ such that the equilibrium half-spread $s^*$
Equilibrium spread and exchange speed

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There exists $T_{\gamma,H}$ such that the equilibrium half-spread $s^*$

1. increases in exchange speed (i.e., decreases in $\delta$) if $\frac{\alpha}{\mu} < T_{\gamma,H}$;
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1. increases in exchange speed (i.e., decreases in $\delta$) if $\frac{\alpha}{\mu} < T_{\gamma,H}$;
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There exists $T_{\gamma,H}$ such that the equilibrium half-spread $s^*$

1. increases in exchange speed (i.e., decreases in $\delta$) if $\frac{\alpha}{\mu} < T_{\gamma,H}$;
2. decreases in exchange speed (i.e., increases in $\delta$) if $\frac{\alpha}{\mu} > T_{\gamma,H}$;
3. does not depend on exchange speed if $\frac{\alpha}{\mu} = T_{\gamma,H}$,
Threshold $T_{\gamma,H}$

![3D graph showing the relationship between the number of HFTs (H) and N/LT threshold ($T$) with varying HFT risk aversion ($\gamma$).]
Latency effect on the equilibrium spread

Bid-ask half spread
high N/LT security

Bid-ask half spread
low N/LT security

Exchange latency ($\delta$)

Faster exchange

Slower exchange
Latency effect on HFT-HFT trade probability

1. HFM-HFB trade probability:

\[
\frac{\mathbb{P}(\text{HFM-HFB trade})}{\mathbb{P}(\text{HFM trade})} = \frac{\frac{\alpha}{\mu + \alpha} \left[ \frac{H-1}{H} \left(1 - \frac{\mu \delta}{2}\right) \right]}{\frac{\alpha}{\mu + \alpha} \left[ \frac{H-1}{H} \left(1 - \frac{\mu \delta}{2}\right) \right] + \frac{\alpha}{\mu + \alpha} \frac{\mu \delta}{2} + \frac{\mu}{\mu + \alpha}}. 
\]
Latency effect on HFT-HFT trade probability

1. HFM-HFB trade probability:

\[
\frac{P(HFM-HFB \, \text{trade})}{P(HFM \, \text{trade})} = \frac{\frac{\alpha}{\mu + \alpha} \left[ \frac{H-1}{H} \left( 1 - \frac{\mu \delta}{2} \right) \right]}{\frac{\alpha}{\mu + \alpha} \left[ \frac{H-1}{H} \left( 1 - \frac{\mu \delta}{2} \right) \right] + \frac{\alpha}{\mu + \alpha} \frac{\mu \delta}{2} + \frac{\mu}{\mu + \alpha}}.
\]

2. HFM-HFB trade probability conditional on news arrival:

\[
\frac{P(HFM-HFB \, \text{trade} — \text{news})}{P(HFM \, \text{trade} — \text{news})} = \frac{\frac{H-1}{H} \left( 1 - \frac{\mu \delta}{2} \right)}{\frac{H-1}{H} \left( 1 - \frac{\mu \delta}{2} \right) + \frac{\mu \delta}{2}}.
\]
Corollary 4

The probability of an HFT-HFT trade increases in exchange speed (i.e., it decreases in $\delta$).
Mixed-sniping equilibrium

Proposition 3
For $\gamma > \bar{\gamma}$ there exist multiple equilibria indexed by the sniping probability of HFBs: $p$. All these equilibria yield the same unique mixed-sniping spread:

$$s^{**} = \sigma \frac{2 - \delta \mu}{2 - \delta \mu + \alpha \delta (\gamma - 1)},$$

where $0 < s^{**} < \sigma$. The strategies that support these equilibria are:

1. At $t = -1$, all HFTs submit one buy limit order at $v_{-1} - s^{**}$ and one sell limit order at $v_{-1} + s^{**}$.
2. If the trigger event at $t = 0$ is a news arrival, then the HFM submits a quote-cancel order. At the same time, with probability $p \leq p^*$, all HFBs submit a market order aimed at the stale quote on the news side of the book (i.e., the ask side if news was good or the bid side when news was bad).
Mixed-sniping equilibrium

Equilibrium half-spread $s^{**}$

Range for HFM equilibrium utility, depending on sniping probability $p$

$HFM$ utility

$HFB$ utility

$p = p^*$

$p = 0$

HFBs

snipe

do not snipe
Maximum sniping probability and exchange latency

Maximum sniping probability ($p^*$)

Mixed-sniping equilibrium

Sure-sniping equilibrium

$\delta = \bar{\delta}_\gamma$

Slower exchange

Faster exchange
Sure- and mixed-sniping equilibria

The diagram illustrates the relationship between exchange latency $\delta$ and HFT risk aversion $\gamma$, highlighting the mixed-sniping equilibrium and the sure-sniping equilibrium.

- \( \gamma \) threshold (H=5)
- \( \gamma \) threshold (H=20)

The shaded area represents the mixed-sniping equilibrium, while the curves indicate the sure-sniping equilibrium for different values of HFT risk aversion and exchange latency.
Mixed-sniping equilibrium

Proposition 4
The mixed-sniping equilibrium half-spread $s^{**}$ increases in exchange speed (i.e., it decreases in $\delta$).
Sure- and mixed-sniping equilibrium spread (high N/LT)

\[ \delta = \bar{\delta}_\gamma \]

- Faster exchange
- Slower exchange
- Mixed-sniping equilibrium
- Sure-sniping equilibrium

Equilibrium bid-ask half-spread
Sure- and mixed-sniping equilibrium spread (low N/LT)

\[ \delta = \bar{\delta} \gamma \]

Faster exchange

Slower exchange

Mixed-sniping equilibrium
Sure-sniping equilibrium

Exchange latency (\(\delta\))
Outline

Motivation

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2. A higher exchange speed makes a high-frequency market-maker duel with high-frequency bandits more often.
3. At the same time, a faster exchange allows the high-frequency market maker to update his quotes more quickly and reduce his payoff risk.
Conclusion

1. Lowering exchange latency can reduce liquidity.
2. A higher exchange speed makes a high-frequency market-maker duel with high-frequency bandits more often.
3. At the same time, a faster exchange allows the high-frequency market maker to update his quotes more quickly and reduce his payoff risk.
4. The net effect on liquidity depends on news-to-liquidity-trader ratio and HFT risk aversion.


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