

Need for Speed? Exchange Latency and Liquidity

Albert J. Menkveld¹ Marius A. Zoican²

¹VU University Amsterdam and Tinbergen Institute

²Université Paris-Dauphine

American Economic Association January 8, 2017

Outline

Motivation

Model

Conclusion

THE WALL STREET JOURNAL. $\equiv |$ business

1 of 12

TOP STORIES IN BUSINESS



In Germany, Amazon Keeps Unions at Bay



• Websites Are Wary of Facebook Tracking

2 of 12



• New to Slow Invers

BUSINESS

NYSE's Fast-Trade Hub Rises Up in New Jersey



LSE goes live with faster trading system

CitiFX launches Velocity 2.0; stakes claim as the fastest platform in market

by Hamish Risk, Laurence Twelvetrees

February 8, 2010

NASDAQ OMX Launches INET Trading System

Question:

Question: Is an even faster exchange good for liquidity?

1. Theory:

HFTs fast/informed speculators...

(Foucault, Hombert, and Roşu, 2015; Biais, Foucault, and Moinas, 2015)

1. Theory:

HFTs fast/informed speculators...

(Foucault, Hombert, and Roşu, 2015; Biais, Foucault, and Moinas, 2015)

... or endogenously become market maker...

(Jovanovic and Menkveld, 2015)

1. Theory:

HFTs fast/informed speculators...

(Foucault, Hombert, and Roșu, 2015; Biais, Foucault, and Moinas, 2015)

... or endogenously become market maker...

(Jovanovic and Menkveld, 2015)

... or are on both sides.

(Budish, Cramton, and Shim, 2015; Haas and Zoican, 2016)

1. Theory:

HFTs fast/informed speculators...

(Foucault, Hombert, and Roșu, 2015; Biais, Foucault, and Moinas, 2015)

... or endogenously become market maker...

(Jovanovic and Menkveld, 2015)

... or are on both sides.

(Budish, Cramton, and Shim, 2015; Haas and Zoican, 2016)

2. Evidence:

HFTs adverse select and get adverse selected. (Hendershott and Riordan, 2011; Baron, Brogaard, and Kirilenko, 2014; Brogaard, Hendershott, and Riordan, 2014).

1. Theory:

HFTs fast/informed speculators...

(Foucault, Hombert, and Roșu, 2015; Biais, Foucault, and Moinas, 2015)

... or endogenously become market maker...

(Jovanovic and Menkveld, 2015)

... or are on both sides.

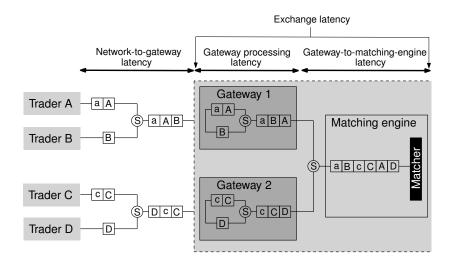
(Budish, Cramton, and Shim, 2015; Haas and Zoican, 2016)

2. Evidence:

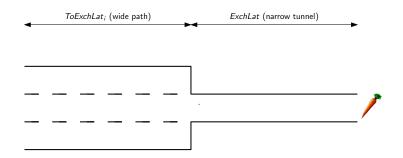
HFTs adverse select and get adverse selected. (Hendershott and Riordan, 2011; Baron, Brogaard, and Kirilenko, 2014; Brogaard, Hendershott, and Riordan, 2014).

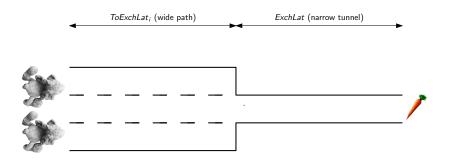
3. Baron, Brogaard, and Kirilenko (2014) and Hagstromer and Norden (2013) find evidence for both market-making and speculative HFT strategies.

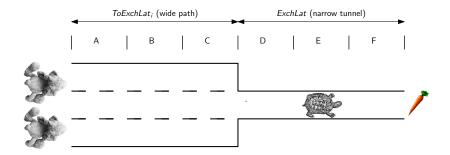
Topology of modern exchanges

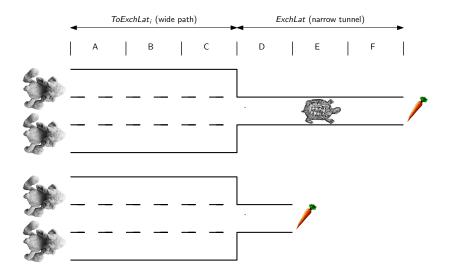


Our attempt on exchange speed and liquidity (in pictures)











1. Lowering exchange latency can reduce liquidity.

Takeaways

- 1. Lowering exchange latency can reduce liquidity.
- 2. A higher exchange speed makes a high-frequency market-maker duel with high-frequency bandits more often.

Takeaways

- 1. Lowering exchange latency can reduce liquidity.
- 2. A higher exchange speed makes a high-frequency market-maker duel with high-frequency bandits more often.
- 3. At the same time, a faster exchange allows the high-frequency market maker to update his quotes more quickly and reduce his payoff risk.

Takeaways

- 1. Lowering exchange latency can reduce liquidity.
- 2. A higher exchange speed makes a high-frequency market-maker duel with high-frequency bandits more often.
- 3. At the same time, a faster exchange allows the high-frequency market maker to update his quotes more quickly and reduce his payoff risk.
- 4. The net effect on liquidity depends on news-to-liquidity-trader ratio and HFT risk aversion.

Outline

Motivation

Model

Conclusion

Agents

1. Informed and fast: *H* high-frequency traders (HFTs).

Agents

 Informed and fast: *H* high-frequency traders (HFTs). HFTs are risk-averse with piece-wise linear utility (Moinas and Pouget, 2013):

$$U^{\mathsf{HFT}}(x) = \gamma x \mathbb{1}_{x < 0} + x \mathbb{1}_{x \ge 0}, \tag{1}$$

Agents

 Informed and fast: *H* high-frequency traders (HFTs). HFTs are risk-averse with piece-wise linear utility (Moinas and Pouget, 2013):

$$U^{\mathsf{HFT}}(x) = \gamma x \mathbb{1}_{x < 0} + x \mathbb{1}_{x \ge 0}, \tag{1}$$

HFTs choose between two strategies:

Agents

 Informed and fast: *H* high-frequency traders (HFTs). HFTs are risk-averse with piece-wise linear utility (Moinas and Pouget, 2013):

$$U^{\mathsf{HFT}}(x) = \gamma x \mathbb{1}_{x < 0} + x \mathbb{1}_{x \ge 0}, \tag{1}$$

HFTs choose between two strategies:

1.1 High-frequency market maker (HFM)

Agents

 Informed and fast: *H* high-frequency traders (HFTs). HFTs are risk-averse with piece-wise linear utility (Moinas and Pouget, 2013):

$$U^{\mathsf{HFT}}(x) = \gamma x \mathbb{1}_{x < 0} + x \mathbb{1}_{x \ge 0}, \tag{1}$$

HFTs choose between two strategies:

- 1.1 High-frequency market maker (HFM)
- 1.2 High-frequency bandit (HFB)

Agents

 Informed and fast: *H* high-frequency traders (HFTs). HFTs are risk-averse with piece-wise linear utility (Moinas and Pouget, 2013):

$$U^{\mathsf{HFT}}(x) = \gamma x \mathbb{1}_{x < 0} + x \mathbb{1}_{x \ge 0}, \tag{1}$$

HFTs choose between two strategies:

- 1.1 High-frequency market maker (HFM)
- 1.2 High-frequency bandit (HFB)

HFTs have an inventory constraint of one unit (long/short).

Agents

 Informed and fast: *H* high-frequency traders (HFTs). HFTs are risk-averse with piece-wise linear utility (Moinas and Pouget, 2013):

$$U^{\mathsf{HFT}}(x) = \gamma x \mathbb{1}_{x < 0} + x \mathbb{1}_{x \ge 0}, \tag{1}$$

HFTs choose between two strategies:

- 1.1 High-frequency market maker (HFM)
- 1.2 High-frequency bandit (HFB)

HFTs have an inventory constraint of one unit (long/short).

2. Uninformed and slow: Liquidity traders (LT).

Agents

 Informed and fast: *H* high-frequency traders (HFTs). HFTs are risk-averse with piece-wise linear utility (Moinas and Pouget, 2013):

$$U^{\mathsf{HFT}}(x) = \gamma x \mathbb{1}_{x < 0} + x \mathbb{1}_{x \ge 0}, \tag{1}$$

HFTs choose between two strategies:

- 1.1 High-frequency market maker (HFM)
- 1.2 High-frequency bandit (HFB)

HFTs have an inventory constraint of one unit (long/short).

2. Uninformed and slow: Liquidity traders (LT).

Exchange

1. Limit order book.

Agents

 Informed and fast: *H* high-frequency traders (HFTs). HFTs are risk-averse with piece-wise linear utility (Moinas and Pouget, 2013):

$$U^{\mathsf{HFT}}(x) = \gamma x \mathbb{1}_{x < 0} + x \mathbb{1}_{x \ge 0}, \tag{1}$$

HFTs choose between two strategies:

- 1.1 High-frequency market maker (HFM)
- 1.2 High-frequency bandit (HFB)

HFTs have an inventory constraint of one unit (long/short).

2. Uninformed and slow: Liquidity traders (LT).

Exchange

- 1. Limit order book.
- 2. Latency: HFTs send messages at t, processed at $t + \delta$.

Asset

1. News arrives with probability $\alpha \tau$ in a period of length τ .

Asset

1. News arrives with probability $\alpha \tau$ in a period of length τ . Common value jumps by $\pm \sigma$.

Asset

- 1. News arrives with probability $\alpha \tau$ in a period of length τ . Common value jumps by $\pm \sigma$.
- 2. LT arrival with probability $\mu \tau$ in a period of length τ .

Asset

- 1. News arrives with probability $\alpha \tau$ in a period of length τ . Common value jumps by $\pm \sigma$.
- 2. LT arrival with probability $\mu \tau$ in a period of length τ . Private value is $\pm \sigma'$, $\sigma' > \sigma$.

Asset

- 1. News arrives with probability $\alpha \tau$ in a period of length τ . Common value jumps by $\pm \sigma$.
- 2. LT arrival with probability $\mu \tau$ in a period of length τ . Private value is $\pm \sigma'$, $\sigma' > \sigma$.
- 3. Either zero or one event possible in a latency interval δ . The probability of two or more events is ignored.

Asset

- 1. News arrives with probability $\alpha \tau$ in a period of length τ . Common value jumps by $\pm \sigma$.
- 2. LT arrival with probability $\mu \tau$ in a period of length τ . Private value is $\pm \sigma'$, $\sigma' > \sigma$.
- 3. Either zero or one event possible in a latency interval δ . The probability of two or more events is ignored.

Common value v_t can change in an interval δ :

Asset

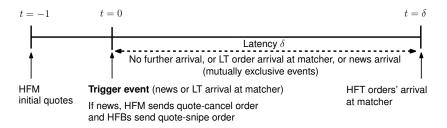
- 1. News arrives with probability $\alpha \tau$ in a period of length τ . Common value jumps by $\pm \sigma$.
- 2. LT arrival with probability $\mu \tau$ in a period of length τ . Private value is $\pm \sigma'$, $\sigma' > \sigma$.
- 3. Either zero or one event possible in a latency interval δ . The probability of two or more events is ignored.

Common value v_t can change in an interval δ :

$$v_{t+\delta} = egin{cases} v_t - \sigma & (ext{``bad'' news arrival}) \ v_t & (ext{no news arrival}) \ v_t + \sigma & (ext{``good'' news arrival}) \end{cases}$$

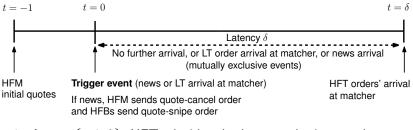
Timing

Timing of the model is as follows:



Timing

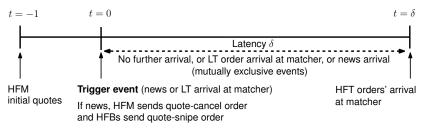
Timing of the model is as follows:



1. At $t \in \{-1, 0\}$, HFTs decide whether to submit a market order, cancel limit orders, or both.

Timing

Timing of the model is as follows:



- 1. At $t \in \{-1, 0\}$, HFTs decide whether to submit a market order, cancel limit orders, or both.
- 2. HFTs arrive at the market in random order. Market orders and cancellations execute, new price quotes are submitted.

1. We search for symmetric Nash equilibria.

- $1. \ \mbox{We search for symmetric Nash equilibria.}$
- 2. All HFTs take same action at t = -1. All HFMs and all HFBs take same action at t = 0.

- 1. We search for symmetric Nash equilibria.
- 2. All HFTs take same action at t = -1. All HFMs and all HFBs take same action at t = 0.

Two equilibrium types

There exists a risk-aversion threshold $\bar{\gamma}$ such that:

- 1. We search for symmetric Nash equilibria.
- 2. All HFTs take same action at t = -1. All HFMs and all HFBs take same action at t = 0.

Two equilibrium types

There exists a risk-aversion threshold $\bar{\gamma}$ such that:

1. For $\gamma \leq \bar{\gamma}$, a *sure-sniping* equilibrium emerges.

- 1. We search for symmetric Nash equilibria.
- 2. All HFTs take same action at t = -1. All HFMs and all HFBs take same action at t = 0.

Two equilibrium types

There exists a risk-aversion threshold $\bar{\gamma}$ such that:

- 1. For $\gamma \leq \bar{\gamma}$, a *sure-sniping* equilibrium emerges.
- 2. For $\gamma > \bar{\gamma}$, a *mixed-sniping* equilibrium emerges.

- 1. We search for symmetric Nash equilibria.
- 2. All HFTs take same action at t = -1. All HFMs and all HFBs take same action at t = 0.

Two equilibrium types

There exists a risk-aversion threshold $\bar{\gamma}$ such that:

- 1. For $\gamma \leq \overline{\gamma}$, a *sure-sniping* equilibrium emerges.
- 2. For $\gamma > \bar{\gamma}$, a *mixed-sniping* equilibrium emerges.

Sniping equilibrium (baseline)

1. In equilibrium, HFTs are indifferent between HFM and HFB strategies.

- 1. We search for symmetric Nash equilibria.
- 2. All HFTs take same action at t = -1. All HFMs and all HFBs take same action at t = 0.

Two equilibrium types

There exists a risk-aversion threshold $\bar{\gamma}$ such that:

- 1. For $\gamma \leq \bar{\gamma}$, a *sure-sniping* equilibrium emerges.
- 2. For $\gamma > \bar{\gamma}$, a *mixed-sniping* equilibrium emerges.

Sniping equilibrium (baseline)

- 1. In equilibrium, HFTs are indifferent between HFM and HFB strategies.
- 2. Equilibrium half-spread s^* nailed by indifference condition:

$$U_{HFM}\left(s^{*}
ight)=U_{HFB}\left(s^{*}
ight).$$

Let $U_{\text{HFB}}(s|\text{trade})$ be the expected HFB profit conditional on a trade (s is half-spread, referred to as spread throughout):

Let $U_{\text{HFB}}(s|\text{trade})$ be the expected HFB profit conditional on a trade (s is half-spread, referred to as spread throughout):

$$U_{\mathsf{HFB}}\left(s|\mathsf{trade}\right) = \underbrace{\left(1 - \frac{\mu\delta}{2} - \alpha\delta\right)(\sigma - s)}_{\mathsf{No event during latency delay}} + \underbrace{\alpha\delta\left[\frac{1}{2}\left(2\sigma - s\right) - \frac{1}{2}s\gamma\right]}_{\mathsf{News arrives during latency delay}}$$

Let $U_{\text{HFB}}(s|\text{trade})$ be the expected HFB profit conditional on a trade (s is half-spread, referred to as spread throughout):

$$U_{\mathsf{HFB}}\left(s|\mathsf{trade}\right) = \underbrace{\left(1 - \frac{\mu\delta}{2} - \alpha\delta\right)(\sigma - s)}_{\mathsf{No event during latency delay}} + \underbrace{\alpha\delta\left[\frac{1}{2}\left(2\sigma - s\right) - \frac{1}{2}s\gamma\right]}_{\mathsf{News arrives during latency delay}}$$

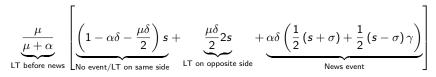
Each HFT is first to the market with probability $\frac{1}{H}$.

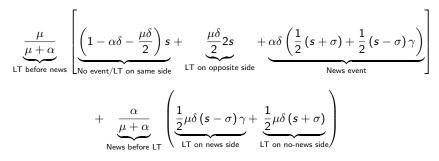
Let $U_{\text{HFB}}(s|\text{trade})$ be the expected HFB profit conditional on a trade (s is half-spread, referred to as spread throughout):

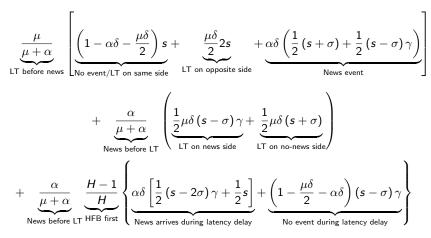
$$U_{\text{HFB}}\left(s|\text{trade}\right) = \underbrace{\left(1 - \frac{\mu\delta}{2} - \alpha\delta\right)(\sigma - s)}_{\text{No event during latency delay}} + \underbrace{\alpha\delta\left[\frac{1}{2}\left(2\sigma - s\right) - \frac{1}{2}s\gamma\right]}_{\text{News arrives during latency delay}}$$

Each HFT is first to the market with probability $\frac{1}{H}$. The HFB expected profit is:

$$U_{\mathsf{HFB}}(s) = \underbrace{\frac{\alpha}{\mu + \alpha}}_{\mathsf{News before LT}} \underbrace{\frac{1}{H}}_{\mathsf{HFB first}} U_{\mathsf{HFB}}(s|\mathsf{trade}).$$







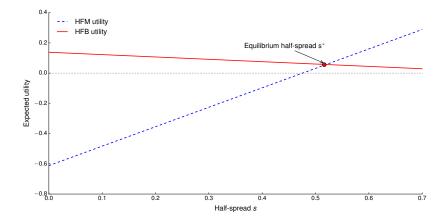
Sure-sniping equilibrium

Proposition 1

The following strategies for HFM and HFB constitute a unique equilibrium for $\gamma<\bar{\gamma}.$

- 1. At t = -1, all HFTs submit one buy limit order at $v_0 s^*$ and one sell limit order at $v_0 + s^*$. The first arriving HFT (picked randomly) fills the order book; we refer to this HFT as the HFM and to the other HFTs as HFBs.
- 2. A trigger event occurs at time t = 0. If the trigger event is a news arrival (i.e., if $v_0 \neq v_{-1}$), then the HFM submits a quote-cancel order and, at the same time, all HFBs submit a market order aimed at the stale quote on the news side of the book (i.e., the ask side if news was good or the bid side when news was bad).

Sure-sniping equilibrium



Equilibrium spread

The equilibrium spread is

$$s^{*} = \sigma \frac{\alpha \left[\delta \mu \left(2+H\right)-2\gamma \left(H+\delta \mu-1\right)-2\right]}{\alpha^{2} \delta \left(\gamma-1\right) \left(H-2\right)-\mu H \left(2+\delta \mu\right)-\alpha \left[2+\delta \mu \left(H-2\right)+2\gamma \left(H-1+\delta \mu\right)\right]}$$

Equilibrium spread

The equilibrium spread is

$$s^{*} = \sigma \frac{\alpha \left[\delta \mu \left(2+H\right)-2\gamma \left(H+\delta \mu-1\right)-2\right]}{\alpha^{2} \delta \left(\gamma-1\right) \left(H-2\right)-\mu H \left(2+\delta \mu\right)-\alpha \left[2+\delta \mu \left(H-2\right)+2\gamma \left(H-1+\delta \mu\right)\right]}$$

1. Comparative statics: $s^* \nearrow \alpha$, $s^* \nearrow \sigma$, $s^* \searrow \mu$, $s^* \nearrow \gamma$.

Equilibrium spread

The equilibrium spread is

$$s^{*} = \sigma \frac{\alpha \left[\delta \mu \left(2+H\right)-2\gamma \left(H+\delta \mu-1\right)-2\right]}{\alpha^{2} \delta \left(\gamma-1\right) \left(H-2\right)-\mu H \left(2+\delta \mu\right)-\alpha \left[2+\delta \mu \left(H-2\right)+2\gamma \left(H-1+\delta \mu\right)\right]}$$

- 1. Comparative statics: $s^* \nearrow \alpha$, $s^* \nearrow \sigma$, $s^* \searrow \mu$, $s^* \nearrow \gamma$.
- Also, equilibrium spread s^{*} ∧ H. More HFBs lead to higher adverse selection costs for the (unique) HFM.

Proposition 2

There exists $T_{\gamma,H}$ such that the equilibrium half-spread s^*

Proposition 2

There exists $T_{\gamma,H}$ such that the equilibrium half-spread s^*

1. increases in exchange speed (i.e., decreases in δ) if $\frac{\alpha}{\mu} < T_{\gamma,H}$;

Proposition 2

There exists $T_{\gamma,H}$ such that the equilibrium half-spread s^*

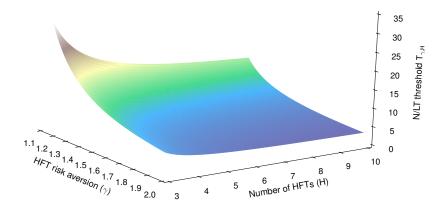
- 1. increases in exchange speed (i.e., decreases in δ) if $\frac{\alpha}{\mu} < T_{\gamma,H}$;
- 2. decreases in exchange speed (i.e., increases in δ) if $\frac{\alpha}{\mu} > T_{\gamma,H}$;

Proposition 2

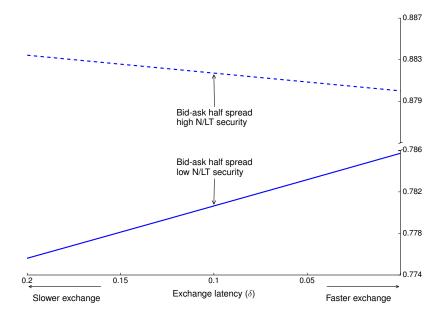
There exists $T_{\gamma,H}$ such that the equilibrium half-spread s^*

- 1. increases in exchange speed (i.e., decreases in δ) if $\frac{\alpha}{\mu} < T_{\gamma,H}$;
- 2. decreases in exchange speed (i.e., increases in δ) if $\frac{\alpha}{\mu} > T_{\gamma,H}$;
- 3. does not depend on exchange speed if $\frac{lpha}{\mu}=T_{\gamma,H}$,

Threshold $T_{\gamma,H}$



Latency effect on the equilibrium spread



Latency effect on HFT-HFT trade probability

1. HFM-HFB trade probability:

$$\frac{\mathbb{P}\left(\mathsf{HFM}\mathsf{-}\mathsf{HFB trade}\right)}{\mathbb{P}\left(\mathsf{HFM trade}\right)} = \frac{\frac{\alpha}{\mu+\alpha} \left[\frac{H-1}{H} \left(1-\frac{\mu\delta}{2}\right)\right]}{\frac{\alpha}{\mu+\alpha} \left[\frac{H-1}{H} \left(1-\frac{\mu\delta}{2}\right)\right] + \frac{\alpha}{\mu+\alpha} \frac{\mu\delta}{2} + \frac{\mu}{\mu+\alpha}}$$

•

Latency effect on HFT-HFT trade probability

1. HFM-HFB trade probability:

$$\frac{\mathbb{P}\left(\mathsf{HFM}\mathsf{-}\mathsf{HFB} \text{ trade}\right)}{\mathbb{P}\left(\mathsf{HFM} \text{ trade}\right)} = \frac{\frac{\alpha}{\mu+\alpha} \left[\frac{H-1}{H} \left(1-\frac{\mu\delta}{2}\right)\right]}{\frac{\alpha}{\mu+\alpha} \left[\frac{H-1}{H} \left(1-\frac{\mu\delta}{2}\right)\right] + \frac{\alpha}{\mu+\alpha} \frac{\mu\delta}{2} + \frac{\mu}{\mu+\alpha}}.$$

2. HFM-HFB trade probability conditional on news arrival:

$$\frac{\mathbb{P}(\mathsf{HFM}\mathsf{-}\mathsf{HFB} \mathsf{ trade} - \mathsf{news})}{\mathbb{P}(\mathsf{HFM} \mathsf{ trade} - \mathsf{news})} = \frac{\frac{H-1}{H}\left(1 - \frac{\mu\delta}{2}\right)}{\frac{H-1}{H}\left(1 - \frac{\mu\delta}{2}\right) + \frac{\mu\delta}{2}}.$$

Latency effect on HFT-HFT trade probability

Corollary 4

The probability of an HFT-HFT trade increases in exchange speed (i.e., it decreases in δ).

Mixed-sniping equilibrium

Proposition 3

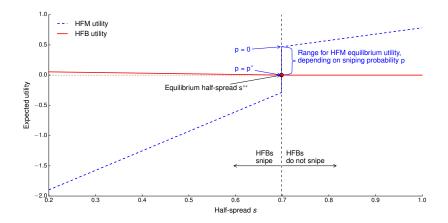
For $\gamma > \overline{\gamma}$ there exist multiple equilibria indexed by the sniping probability of HFBs: *p*. All these equilibria yield the same unique mixed-sniping spread:

$$s^{**} = \sigma \frac{2 - \delta \mu}{2 - \delta \mu + \alpha \delta \left(\gamma - 1\right)},\tag{2}$$

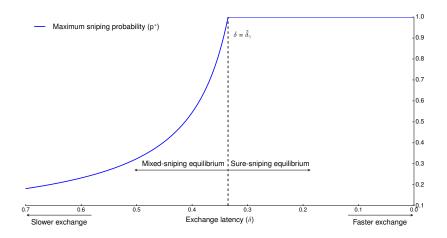
where $0 < s^{**} < \sigma$. The strategies that support these equilibria are:

- 1. At t = -1, all HFTs submit one buy limit order at $v_{-1} s^{**}$ and one sell limit order at $v_{-1} + s^{**}$.
- 2. If the trigger event at t = 0 is a news arrival, then the HFM submits a quote-cancel order. At the same time, with probability $p \le p^*$, all HFBs submit a market order aimed at the stale quote on the news side of the book (i.e., the ask side if news was good or the bid side when news was bad).

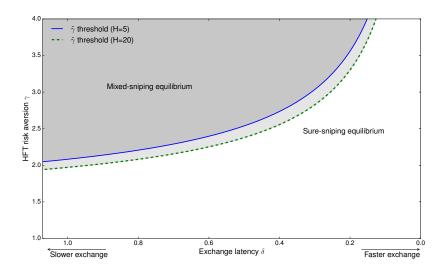
Mixed-sniping equilibrium



Maximum sniping probability and exchange latency



Sure- and mixed-sniping equilibria

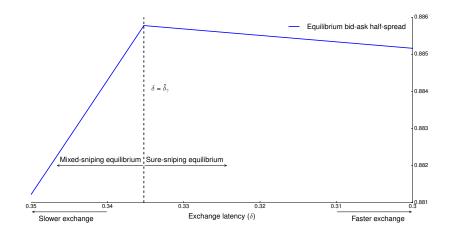


Mixed-sniping equilibrium

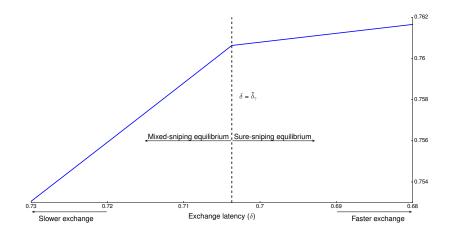
Proposition 4

The mixed-sniping equilibrium half-spread s^{**} increases in exchange speed (i.e., it decreases in δ).

Sure- and mixed-sniping equilibrium spread (high N/LT)



Sure- and mixed-sniping equilibrium spread (low N/LT)



Outline

Motivation

Model

1. Lowering exchange latency can reduce liquidity.

- 1. Lowering exchange latency can reduce liquidity.
- 2. A higher exchange speed makes a high-frequency market-maker duel with high-frequency bandits more often.

- 1. Lowering exchange latency can reduce liquidity.
- 2. A higher exchange speed makes a high-frequency market-maker duel with high-frequency bandits more often.
- 3. At the same time, a faster exchange allows the high-frequency market maker to update his quotes more quickly and reduce his payoff risk.

- 1. Lowering exchange latency can reduce liquidity.
- 2. A higher exchange speed makes a high-frequency market-maker duel with high-frequency bandits more often.
- 3. At the same time, a faster exchange allows the high-frequency market maker to update his quotes more quickly and reduce his payoff risk.
- 4. The net effect on liquidity depends on news-to-liquidity-trader ratio and HFT risk aversion.

- Baron, Matthew, Jonathan Brogaard, and Andrei A. Kirilenko, 2014, Risk and return in High Frequency Trading, *Working paper*.
- Biais, Bruno, Thierry Foucault, and Sophie Moinas, 2015, Equilibrium fast trading, *Journal of Financial Economics* 116, 292–313.
- Breckenfelder, Johannes, 2013, Competition between high-frequency traders, and market quality, *Working paper*.
- Brogaard, Jonathan, Terrence Hendershott, and Ryan Riordan, 2014, High frequency trading and price discovery, *Review of Financial Studies* 27, 2267–2306.
- Budish, Eric, Peter Cramton, and John Shim, 2015, The high-frequency trading arms race: Frequent batch auctions as a market design response, *Working paper*, *University of Chicago*.
- Foucault, Thierry, Johan Hombert, and Ioanid Roşu, 2015, News trading and speed, *Journal of Finance, forthcoming*.
- Haas, Marlene D., and Marius A. Zoican, 2016, Discrete or continuous trading?HFT competition and liquidity on batch auction markets, *Working paper*.Hagstromer, Björn, and Lars Norden, 2013, The diversity of high-frequency
- traders, Journal of Financial Markets 16, 741–770.
- Hendershott, T., and R. Riordan, 2011, High-frequency trading and price discovery, *working paper*.
- Jovanovic, Boyan, and Albert J. Menkveld, 2015, Middlemen in limit-order markets, *WFA 2011 paper*.
- Moinas, Sophie, and Sebastien Pouget, 2013, The bubble game: An experimental study of speculation, *Econometrica* 81, 1507–1539.