IT'S NOT MY FAULT—THE GAS PEDAL'S STUCK...
Crowded Risk as a Systemic Concern for Central Clearing Counterparties

Albert J. Menkveld

VU Amsterdam and Tinbergen Institute

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Outline

Motivation

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Appendix
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2. ESRB annual report 2012, p. 16:
   2.1 Structural reforms . . . improved risk management throughout the financial system. In particular, the mandatory move to clearing standardised over-the-counter (OTC) derivatives trades via CCPs will help to reduce counterparty risk between financial institutions . . .
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   2.2 However, the more prominent role of CCPs will also introduce new systemic risks. Mandatory clearing will turn CCPs into systemic nodes in the financial system, with unknown, but possibly far-reaching, consequences.
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   1.2 BIS-IOSCO (2012)  
       “Principles for Financial Market Infrastructures.”
Exhibit 1: 
CPSS-IOSCO Technical Committee 
Recommendations for Central Counterparties (CCPs)

1. **Legal risk**
   
   A CCP should have a well founded, transparent and enforceable legal framework for each aspect of its activities in all relevant jurisdictions.

2. **Participation requirements**
   
   A CCP should require participants to have sufficient financial resources and robust operational capacity to meet obligations arising from participation in the CCP. A CCP should have procedures in place to monitor that participation requirements are met on an ongoing basis. A CCP’s participation requirements should be objective, publicly disclosed, and permit fair and open access.

3. **Measurement and management of credit exposures**
   
   A CCP should measure its credit exposures to its participants at least once a day. Through margin requirements, other risk control mechanisms or a combination of both, a CCP should limit its exposures to potential losses from defaults by its participants in normal market conditions so that the operations of the CCP would not be disrupted and non-defaulting participants would not be exposed to losses that they cannot anticipate or control.

4. **Margin requirements**
   
   If a CCP relies on margin requirements to limit its credit exposures to participants, those requirements should be sufficient to cover potential exposures in normal market conditions. The models and parameters used in setting margin requirements should be risk-based and reviewed regularly.
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2. A member’s margin scales with his outstanding trades times volatility.
3. For example, 54 exchanges and clearing houses use SPAN developed by Chicago Mercantile Exchange (CME).
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3. And, is there a natural way to allocate CCP exposure across members (according to the “polluter pays” principle)?
Objective

1. Do crowded trades raise CCP exposure toward its members in a way not accounted for by member specific margins? Yes!
2. If so, can one come up with a reasonable measure of crowding? Yes!
3. And, is there a natural way to allocate CCP exposure across members (according to the “polluter pays” principle)? Yes!
Findings

1. CCP exposure is measured by tail risk in aggregate *loss* across all member portfolios with several appealing properties.
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   3.1 \textit{Crowdlx}: A crowding index to measure crowded-trade risk.
   3.2 \textit{ExpCCP}: A CCP exposure measure with decomposition across members (polluter pays).
1. CCP literature

2. Systemic risk literature
   2.3 **Systemic risk measurement/allocation**: Bisias et al. (2012), Brunnermeier and Cheridito (2014), Capponi (2016).
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2. $n_j$ is the vector of yet-to-settle trade portfolio of member $j$.

3. Let $X_j = n_j^t R$ be the P&L on member $j$’s trade portfolio, then

$$ X \sim N(0, \Sigma), \quad \Sigma = N^t \Omega N, \quad N = [n_1, \cdots, n_J]. $$
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\[ X \sim N(0, \Sigma), \quad \Sigma = N' \Omega N, \quad N = [n_1, \ldots, n_J]. \]

4. CCP exposure to trade portfolios of all members is defined as

\[ E(A) = E \left( \sum_j E_j \right) \text{ with } E_j = -\min(X_j, 0). \]
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**Definition**

Let $\text{ExpCCP}$ be the total margin a CCP should collect to protect against tail risk:

$$
\text{ExpCCP} := E(A) + \alpha \text{std}(A).
$$
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**Definition**

Let $\text{ExpCCP}$ be the total margin a CCP should collect to protect against tail risk:

$$\text{ExpCCP} := E(A) + \alpha \text{std}(A).$$

$$E(A) = \sum_j \sqrt{\frac{1}{2\pi}} \sigma_j$$ (Duffie and Zhu, 2011), but what about $\text{std}(A)$?
Absolute Moments in 2-dimensional Normal Distribution

By Seiji Nabeya

Let $x$ and $y$ be distributed according to the following 2-dimensional normal distribution,

$$
\frac{1}{2\pi\sigma_1\sigma_2\sqrt{1-\rho^2}} \exp \left\{ -\frac{1}{2(1-\rho^2)} \left( \frac{x^2}{\sigma_1^2} - \frac{2\rho xy}{\sigma_1\sigma_2} + \frac{y^2}{\sigma_2^2} \right) \right\} dx\,dy.
$$

It is our purpose to express absolute moments in terms of elementary functions. Putting $E(|x^m y^n|) = (m, n)$ for simplicity, we have

$$(m, n) = \frac{1}{2\pi\sigma_1\sigma_2\sqrt{1-\rho^2}} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} |x^m y^n| \exp \left\{ -\frac{1}{2(1-\rho^2)} \left( \frac{x^2}{\sigma_1^2} - 2\rho \frac{xy}{\sigma_1\sigma_2} + \frac{y^2}{\sigma_2^2} \right) \right\} dx\,dy

= \frac{2^{\frac{m+n}{2}}}{\pi} \sigma_1^m \sigma_2^n (1-\rho^2)^{\frac{m+n+1}{2}} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} |x^m y^n| \exp \left( -x^2 + 2\rho xy - y^2 \right) dx\,dy

= \frac{2^{\frac{m+n}{2}}}{\pi} \sigma_1^m \sigma_2^n (1-\rho^2)^{\frac{m+n+1}{2}} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} |x^m y^n| e^{-x^2 - y^2} \sum_{k=0}^{\infty} \frac{(2\rho xy)^k}{k!} dx\,dy

= \frac{2^{\frac{m+n}{2}}}{\pi} \sigma_1^m \sigma_2^n (1-\rho^2)^{\frac{m+n+1}{2}} \sum_{k=0}^{\infty} \frac{\Gamma\left(\frac{m+1}{2} + k\right) \Gamma\left(\frac{n+1}{2} + k\right)}{(2k)!} (2\rho)^k

= \frac{2^{\frac{m+n}{2}}}{\pi} \sigma_1^m \sigma_2^n \Gamma\left(\frac{m+1}{2}\right) \Gamma\left(\frac{n+1}{2}\right) (1-\rho^2)^{\frac{m+n+1}{2}}

\times F\left(\frac{m+1}{2}, \frac{n+1}{2}; \frac{1}{2}; \rho^2\right)

= \frac{2^{\frac{m+n}{2}}}{\pi} \sigma_1^m \sigma_2^n \Gamma\left(\frac{m+1}{2}\right) \Gamma\left(\frac{n+1}{2}\right) F\left(-\frac{m}{2}, -\frac{n}{2}; \frac{1}{2}; \rho^2\right).

Here

$$
F(\alpha, \beta; \gamma; z) = 1 + \frac{\alpha\beta}{1!\gamma} z + \frac{\alpha(\alpha+1)\beta(\beta+1)}{2!\gamma(\gamma+1)} z^2 + \ldots
$$

is the hypergeometric function, which reduces to the polynomial of $z$ if $\alpha$ or $\beta$ is a non-positive integer and $\gamma$ is positive. Thus, when at least one of the integers $m$, $n$ is an even number, $(m, n)$ reduces to the polynomial of $\rho^2$ multiplied by $\sigma_1^m \sigma_2^n$. 
The case where both $m$ and $n$ are odd may be treated as follows. Put

$$z = \sqrt{2(1 - \rho^2)} \sigma_1 \cos \theta, \quad y = \sqrt{2(1 - \rho^2)} \sigma_2 \sin \theta.$$

When $m - n = 2q$, where $q$ is a non-negative integer, we have then

$$(m, n) = \frac{1}{2\pi \sigma_1 \sigma_2 \sqrt{1 - \rho^2}} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} x^my^n |\exp\left(-\frac{r^2}{2(1 - \rho^2)}\right)|
\times \left(\frac{x^2}{\sigma_1^2} - 2\rho \frac{xy}{\sigma_1 \sigma_2} + \frac{y^2}{\sigma_2^2}\right) dx dy$$

$$= \frac{2^{m+n} \pi^{m-n}}{\pi} (1 - \rho^2)^{\frac{m+n+1}{2}} \int_{0}^{2\pi} \int_{0}^{\infty} r^{m+n+1} |\cos^m \theta \sin^n \theta|$$
$$\times \exp\left(-r^2(1 - 2\rho \cos \theta \sin \theta)\right) dr d\theta$$

$$= \frac{2^{m-n} \pi^{m-n}}{\pi} \Gamma\left(\frac{m-n}{2} + 1\right) (1 - \rho^2)^{\frac{m+n+1}{2}}$$
$$\times \int_{0}^{2\pi} \left(\frac{\cos^m \theta \sin^n \theta}{(1 - 2\rho \cos \theta \sin \theta)^{\frac{m+n+1}{2}}} + \frac{\cos^m \theta \sin^n \theta}{(1 + 2\rho \cos \theta \sin \theta)^{\frac{m+n+1}{2}}}\right) d\theta$$

$$= \frac{2^{m-n} \pi^{m-n}}{\pi} \Gamma\left(\frac{m-n}{2} + 1\right) (1 - \rho^2)^{\frac{m+n+1}{2}}$$
$$\times \frac{d^n}{dp^n} \int_{0}^{\pi} \frac{\cos^{2n} \theta}{(1 - 2\rho \cos \theta \sin \theta)^{n+1}} - \frac{\cos^{2n} \theta}{(1 + 2\rho \cos \theta \sin \theta)^{n+1}} d\theta.$$  

As the last integral may be calculated in the elementary fashion, $(m, n)$ may be evaluated.

In the following we shall give the obtained formulae for the cases $m \geq n$. The formula of $(m, n)$ for $m \leq n$, is obtained by exchanging $\sigma_1$ and $\sigma_2$ in the formula $(n, m)$.

$$(1, 0) = \frac{2}{\pi} \sigma_1,$$

$$(2, 0) = \sigma_1^2,$$

$$(1, 1) = \frac{2}{\pi} \left(\sqrt{1 - \rho^2} + \rho \sin^{-1} \rho\right) \sigma_1 \sigma_2,$$
\[(3, 0) = 2 \sqrt{\frac{2}{\pi}} \sigma_i^3,\]
\[(2, 1) = \sqrt{\frac{2}{\pi}} \left(1 + \rho^2\right) \sigma_i^3 \sigma_z,\]
\[(4, 0) = 3 \sigma_i^4,\]
\[(3, 1) = \frac{2}{\pi} \left(\sqrt{1 - \rho^2} \left(2 + \rho^4\right) + 3 \rho \sin^{-1} \rho\right) \sigma_i^5 \sigma_z,\]
\[(2, 2) = (1 + 2 \rho^2) \sigma_i^5 \sigma_z,\]
\[(5, 0) = 8 \sqrt{\frac{2}{\pi}} \sigma_i^5,\]
\[(4, 1) = \sqrt{\frac{2}{\pi}} \left(3 + 6 \rho^3 - \rho^4\right) \sigma_i^5 \sigma_z,\]
\[(3, 2) = 2 \sqrt{\frac{2}{\pi}} \left(1 + 3 \rho^3\right) \sigma_i^5 \sigma_z,\]
\[(6, 0) = 15 \sigma_i^6,\]
\[(5, 1) = \frac{2}{\pi} \left(\sqrt{1 - \rho^2} \left(8 + 9 \rho^3 - 2 \rho^4\right) + 15 \rho \sin^{-1} \rho\right) \sigma_i^5 \sigma_z,\]
\[(4, 2) = 3 \left(1 + 4 \rho^2\right) \sigma_i^5 \sigma_z,\]
\[(3, 3) = \frac{2}{\pi} \left(\sqrt{1 - \rho^2} \left(4 + 11 \rho^3\right) + 3 \rho \left(3 + 2 \rho^3\right) \sin^{-1} \rho\right) \sigma_i^5 \sigma_z,\]
\[(7, 0) = 48 \sqrt{\frac{2}{\pi}} \sigma_i^7,\]
\[(6, 1) = 3 \sqrt{\frac{2}{\pi}} \left(5 + 15 \rho^2 - 5 \rho^4 + \rho^6\right) \sigma_i^6 \sigma_z,\]
\[(5, 2) = 8 \sqrt{\frac{2}{\pi}} \left(1 + 5 \rho^2\right) \sigma_i^6 \sigma_z,\]
\[(4, 3) = 6 \sqrt{\frac{2}{\pi}} \left(1 + 6 \rho^2 + \rho^4\right) \sigma_i^6 \sigma_z,\]
\[(7, 1) = \frac{2}{\pi} \left(\sqrt{1 - \rho^2} \left(48 + 87 \rho^2 - 38 \rho^4 + 8 \rho^6\right) + 105 \rho \sin^{-1} \rho\right) \sigma_i^5 \sigma_z,\]
\[(6, 2) = 15 \left(1 + 6 \rho^2\right) \sigma_i^6 \sigma_z,\]
\[(5, 3) = \frac{2}{\pi} \left(\sqrt{1 - \rho^2} \left(16 + 83 \rho^2 + 6 \rho^4\right) + 15 \rho \left(3 + 4 \rho^3\right) \sin^{-1} \rho\right) \sigma_i^5 \sigma_z,\]
\[(4, 4) = 3 \left(3 + 24 \rho^3 + 8 \rho^6\right) \sigma_i^4 \sigma_z,\]
(9, 0) = 384 \sqrt{\frac{2}{\pi}} \sigma_1^2,

(8, 1) = 3 \sqrt{\frac{2}{\pi}} (35 + 140\rho^3 - 70\rho^4 + 28\rho^6 - 5\rho^9) \sigma_1^8 \sigma_2,

(7, 2) = 48 \sqrt{\frac{2}{\pi}} (1 + 7\rho^3) \sigma_1^7 \sigma_2^2,

(6, 3) = 6 \sqrt{\frac{2}{\pi}} (5 + 45\rho^3 + 15\rho^4 - \rho^6) \sigma_1^6 \sigma_2^3,

(5, 4) = 24 \sqrt{\frac{2}{\pi}} (1 + 10\rho^3 + 5\rho^4) \sigma_1^5 \sigma_2^4,

(10, 0) = 945\sigma_1^m,

(9, 1) = \frac{6}{\pi} \left( \sqrt{1 - \rho^2} (128 + 325\rho^3 - 210\rho^4 + 88\rho^6 - 16\rho^9) + 315\rho \sin^{-1}\rho \right) \sigma_1^8 \sigma_2,

(8, 2) = 105(1 + 8\rho^3) \sigma_1^7 \sigma_2^2,

(7, 3) = \frac{2}{\pi} \left( \sqrt{1 - \rho^2} (96 + 741\rho^3 + 120\rho^4 - 12\rho^6) + 315\rho(1 + 2\rho^3) \sin^{-1}\rho \right) \sigma_1^7 \sigma_2^3,

(6, 4) = 45(1 + 12\rho^3 + 8\rho^4) \sigma_1^6 \sigma_2^4,

(5, 5) = \frac{2}{\pi} \left( \sqrt{1 - \rho^2} (64 + 607\rho^3 + 274\rho^4) + 15\rho(15 + 40\rho^3 + 8\rho^4) \sin^{-1}\rho \right) \sigma_1^7 \sigma_2^5,

(11, 0) = 3840 \sqrt{\frac{2}{\pi}} \sigma_1^{11},

(10, 1) = 15 \sqrt{\frac{2}{\pi}} (63 + 315\rho^3 - 210\rho^4 + 126\rho^6 - 45\rho^8 + 7\rho^{10}) \sigma_1^6 \sigma_2,

(9, 2) = 384 \sqrt{\frac{2}{\pi}} (1 + 9\rho^3) \sigma_1^8 \sigma_2^2,

(8, 3) = 6 \sqrt{\frac{2}{\pi}} (35 + 420\rho^3 + 210\rho^4 - 28\rho^6 + 3\rho^9) \sigma_1^7 \sigma_2^3,

(7, 4) = 48 \sqrt{\frac{2}{\pi}} (3 + 42\rho^3 + 35\rho^4) \sigma_1^6 \sigma_2^4,

(6, 5) = 120 \sqrt{\frac{2}{\pi}} (1 + 15\rho^3 + 15\rho^4 + \rho^6) \sigma_1^6 \sigma_2^5,
(12, 0) = 10395\sigma_1^n,

(11, 1) = \frac{6}{\pi} \left\{ \sqrt{1 - \rho^2} (1280 + 4215\rho^2 - 3590\rho^4 + 2248\rho^6 - 816\rho^8 \\
+ 128\rho^{10}) + 3465\rho \sin^{-1}\rho \right\} \sigma_1^n \sigma_2^n,

(10, 2) = 945(1 + 10\rho^2)\sigma_1^n \sigma_2^n,

(9, 3) = \frac{6}{\pi} \left\{ \sqrt{1 - \rho^2} (256 + 2689\rho^2 + 690\rho^4 - 136\rho^6 + 16\rho^8) \\
+ 315\rho(3 + 8\rho^2) \sin^{-1}\rho \right\} \sigma_1^n \sigma_3^n,

(8, 4) = 315(1 + 16\rho^2 + 16\rho^4)\sigma_1^n \sigma_4^n,

(7, 5) = \frac{6}{\pi} \left\{ \sqrt{1 - \rho^2} (128 + 1779\rho^2 + 1518\rho^4 + 40\rho^6) \\
+ 105\rho(5 + 20\rho^2 + 8\rho^4) \sin^{-1}\rho \right\} \sigma_1^n \sigma_5^n,

(6, 6) = 45(5 + 90\rho^2 + 120\rho^4 + 16\rho^6)\sigma_1^n \sigma_6^n.

In another paper we shall treat the 3-dimensional case by a unified but more complicated method.

_Institute of Statistical Mathematics._
CCP exposure

1. Some algebra using results for folded and truncated normal (Nabeya, 1951; Rosenbaum, 1961) yields:

\[
\text{std}(A) = \sqrt{\sum_{k,l} \left( \frac{\pi - 1}{2\pi} \right) \sigma_k \sigma_l M(\rho_{kl})}
\]
CCP exposure

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\[
\text{std}(A) = \sqrt{\sum_{k,l} \left( \frac{\pi - 1}{2\pi} \right) \sigma_k \sigma_l M(\rho_{kl})}
\]

\[
M(\rho) = \left[ \frac{1}{2} \pi + \arcsin(\rho) \right] \rho + \frac{\sqrt{1 - \rho^2} - 1}{\pi - 1}
\]
CCP exposure

Return correlation

Loss correlation

M(.)
Simple example
Noncrowded trades

security/

risk factor 2

security/

risk factor 1
Simple example noncrowded trades

1. 

\[ N = \begin{pmatrix} 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 \end{pmatrix}, \quad \Sigma = \begin{pmatrix} 1 & -1 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & -1 & 1 \end{pmatrix} \]
Simple example noncrowded trades

1. 

\[
N = \begin{pmatrix} 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 \end{pmatrix}, \quad \Sigma = \begin{pmatrix} 1 & -1 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & -1 & 1 \end{pmatrix}
\]

2. 

\[
E(E) = \sqrt{\frac{1}{2\pi}} \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}, \quad \text{var}(E) = \frac{1}{2\pi} \begin{pmatrix} \pi - 1 & -1 & 0 & 0 \\ -1 & \pi - 1 & 0 & 0 \\ 0 & 0 & \pi - 1 & -1 \\ 0 & 0 & -1 & \pi - 1 \end{pmatrix}
\]
Simple example noncrowded trades

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\[ N = \begin{pmatrix} 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 \end{pmatrix}, \quad \Sigma = \begin{pmatrix} 1 & -1 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & -1 & 1 \end{pmatrix} \]

2.

\[ E(E) = \sqrt{\frac{1}{2\pi}} \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}, \quad \text{var}(E) = \frac{1}{2\pi} \begin{pmatrix} \pi - 1 & -1 & 0 & 0 \\ -1 & \pi - 1 & 0 & 0 \\ 0 & 0 & \pi - 1 & -1 \\ 0 & 0 & -1 & \pi - 1 \end{pmatrix} \]

3.

\[ E(A) = 4 \sqrt{\frac{1}{2\pi}} \approx 1.60 \quad \text{and} \quad \text{std}(A) = 2 \sqrt{\frac{\pi - 2}{2\pi}} \approx 0.85 \]
Crowded trades

security/

risk factor 2

security/

risk factor 1

n1

n2

n3

n4
Simple example crowded trades

1.

\[ N = \begin{pmatrix} 1 & -1 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad \Sigma = N'\Omega N = \begin{pmatrix} 1 & -1 & 1 & -1 \\ -1 & 1 & -1 & 1 \\ 1 & -1 & 1 & -1 \\ -1 & 1 & -1 & 1 \end{pmatrix} \]
Simple example crowded trades

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\[ N = \begin{pmatrix} 1 & -1 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad \Sigma = N' \Omega N = \begin{pmatrix} 1 & -1 & 1 & -1 \\ -1 & 1 & -1 & 1 \\ 1 & -1 & 1 & -1 \\ -1 & 1 & -1 & 1 \end{pmatrix} \]

2.

\[ E(E) = \sqrt{\frac{1}{2\pi}} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \quad \text{var}(E) = \frac{1}{2\pi} \begin{pmatrix} \pi - 1 & -1 & \pi - 1 & -1 \\ -1 & \pi - 1 & -1 & \pi - 1 \\ \pi - 1 & -1 & \pi - 1 & -1 \\ -1 & \pi - 1 & -1 & \pi - 1 \end{pmatrix} \]
Simple example crowded trades

1. 
\[ N = \begin{pmatrix} 1 & -1 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad \Sigma = N'\Omega N = \begin{pmatrix} 1 & -1 & 1 & -1 \\ -1 & 1 & -1 & 1 \\ 1 & -1 & 1 & -1 \\ -1 & 1 & -1 & 1 \end{pmatrix} \]

2. 
\[ E(E) = \sqrt{\frac{1}{2\pi}} \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}, \quad \text{var}(E) = \frac{1}{2\pi} \begin{pmatrix} \pi - 1 & -1 & \pi - 1 & -1 \\ -1 & \pi - 1 & -1 & \pi - 1 \\ \pi - 1 & -1 & \pi - 1 & -1 \\ -1 & \pi - 1 & -1 & \pi - 1 \end{pmatrix} \]

3. 
\[ E(A) = 4 \sqrt{\frac{1}{2\pi}} \approx 1.60 \quad \text{and} \quad \text{std}(A) = 2 \sqrt{\frac{\pi - 2}{\pi}} \approx 1.21 \]
Histogram CCP exposure (N=4)

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- **Illustration**
- **Conclusion**
- **Appendix**
- **References**
Two tools
Tool #1: CrowdIx

Is there a natural “thermometer” for crowded-trade risk?

1 A feasible approach to this NP hard problem is to convert it to a standard bin-packing problem which can be “solved” heuristically (see Appendix A of the slides).
Tool #1: CrowdIx

Is there a natural “thermometer” for crowded-trade risk?

Definition

CrowdIx for $\Sigma$ is defined as

$$CrowdIx = \frac{\text{std}(A)}{\text{std}(\tilde{A})}$$

where $\tilde{A}$ is CCP aggregate exposure when all members’ trades are re-allocated to a single risk factor to the maximum extent possible.\(^1\)

\(^1\) A feasible approach to this NP hard problem is to convert it to a standard bin-packing problem which can be “solved” heuristically (see Appendix A of the slides).
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Lemma

$$\text{CrowdIx} \geq \sqrt[2]{\frac{1}{J/2}} \quad \text{where} \quad \tilde{J} = 2 \lfloor J/2 \rfloor J$$

\(^1\) A feasible approach to this NP hard problem is to convert it to a standard bin-packing problem which can be “solved” heuristically (see Appendix A of the slides).
Tool #1: CrowdIx

1. CrowdIx in the simple example is
\[
\begin{cases}
\sqrt{1/2} = 0.71 & \text{in the noncrowded case.} \\
1 & \text{in the crowded case.}
\end{cases}
\]
Tool #2: ExpCCP

1. Homogeneity of degree one of \( E(A) \) and \( \text{std}(A) \) implies that ExpCCP naturally decomposes across members (Euler’s homogeneous function theorem).

1.1

\[
E(A) = \sum_j \sqrt{\frac{1}{2\pi}} \sigma_j
\]

1.2

\[
\text{std}(A) = \sum_k \sigma_k \frac{\partial \text{std}(A)}{\partial \sigma_k} = \sum_k \sigma_k \sum_l \frac{1}{\text{std}(A)} \left( \frac{\pi - 1}{2\pi} \right) \sigma_l M(\rho_{kl})
\]
Tool #2: ExpCCP

1. Homogeneity of degree one of $E(A)$ and $\text{std}(A)$ implies that $\text{ExpCCP}$ naturally decomposes across members (Euler’s homogeneous function theorem).

   1.1

   $$E(A) = \sum_j \sqrt{\frac{1}{2\pi}} \sigma_j$$

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   $$\text{std}(A) = \sum_k \sigma_k \frac{\partial \text{std}(A)}{\partial \sigma_k} = \sum_k \sigma_k \sum_l \frac{1}{\text{std}(A)} \left( \frac{\pi - 1}{2\pi} \right) \sigma_l M(\rho_{kl})$$

2. Therefore $\text{ExpCCP}$ equals,

$$\sum_k \sigma_k \left[ \sqrt{\frac{1}{2\pi}} + \frac{\alpha}{\text{std}(A)} \left( \frac{\pi - 1}{2\pi} \right) \sigma_k + \sum_{l \neq k} \frac{\alpha}{\text{std}(A)} \left( \frac{\pi - 1}{2\pi} \right) \sigma_l M(\rho_{kl}) \right].$$

Member-specific part ("old")

Crowded-trade part ("new")
Tool #2: ExpCCP

1. To identify risk factor(s) on which members’ trades crowd, the following results are useful:

1.1

\[
\frac{\partial}{\partial \sigma^f} E(A) = \sum_j \sqrt{\frac{1}{2\pi \sigma_j}} \sigma_f B_{jj}
\]

1.2

\[
\frac{\partial}{\partial \sigma^f} \text{std}(A) = \left( \frac{\pi - 1}{4\pi} \right) \frac{\sigma_f}{\sigma^A} \sum_{k,l} \left[ M'(\rho_{kl})B_{kl} + \rho_{kl}^2 \left( 1 - 2 \sqrt{1 - \rho_{kl}^2} \right) \left( \frac{\sigma_l}{\sigma_k} B_{kk} + \frac{\sigma_k}{\sigma_l} B_{ll} \right) \right]
\]

with

\[B_{kl} := n_k' \beta \beta' n_l \quad \text{and} \quad \beta = \text{cov}(R, r^f)/\text{var}(r^f)\]
Tool #2: $\text{ExpCCP}$

1. The sensitivity of $\text{ExpCCP}$ to a particular risk factor is naturally described by the following elasticity:

$$e_{\sigma_f}^{\text{ExpCCP}} = \frac{\sigma_f}{\text{ExpCCP}} \left( \frac{\partial}{\partial \sigma_f} E(A) + \alpha \frac{\partial}{\partial \sigma_f} \text{std}(A) \right).$$
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References
Data

1. A European Multilateral Clearing Facility (EMCF) sample of trade reports filed by its (anonymized) members.
Data

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Data

1. A European Multilateral Clearing Facility (EMCF) sample of trade reports filed by its (anonymized) members.
2. It contains all trades in stocks listed in Denmark, Finland, and Sweden.
4. It spans almost all exchanges: NASDAQ-OMX, Chi-X, Bats, Burgundy, and Quote MTF (Turquoise not included).
5. Sample consists of 1.4 million trades by 57 clearing members in 242 securities across 228 days.
## Clearing members

<table>
<thead>
<tr>
<th>Bank Name</th>
<th>Phone</th>
</tr>
</thead>
<tbody>
<tr>
<td>ABN AMRO Clearing Bank N.V.</td>
<td>Numis Securities Ltd</td>
</tr>
<tr>
<td>BNP Paribas Securities Services S.A.</td>
<td>UBS Ltd</td>
</tr>
<tr>
<td>Bank of America Merrill Lynch</td>
<td>Barclays Capital Securities Ltd.</td>
</tr>
<tr>
<td>Citibank Global Markets and Citibank International</td>
<td>Alandsbanken Abp</td>
</tr>
<tr>
<td>JPMorgan Securities Ltd.</td>
<td>Alandsbanken Sverige AB</td>
</tr>
<tr>
<td>Goldman Sachs International</td>
<td>Amagarbanken A/S</td>
</tr>
<tr>
<td>Skandinaviska Enskilda Banken</td>
<td>Arbejdernes Landsbank A/S</td>
</tr>
<tr>
<td>KAS BANK N.V.</td>
<td>Avanza Bank AB</td>
</tr>
<tr>
<td>Parel S.A.</td>
<td>Carnegie Bank A/S</td>
</tr>
<tr>
<td>Deutsche Bank AG</td>
<td>Dexia Securities France</td>
</tr>
<tr>
<td>Citigroup</td>
<td>E-Trade Bank</td>
</tr>
<tr>
<td>MF Global UK Ltd</td>
<td>Eik Bank A/S</td>
</tr>
<tr>
<td>CACEIS Bank Deutschland</td>
<td>EQ Bank Ltd.</td>
</tr>
<tr>
<td>Danske Bank</td>
<td>Evli Bank Plc</td>
</tr>
<tr>
<td>ABG Sundal Collier Norge</td>
<td>FIM Bank Ltd.</td>
</tr>
<tr>
<td>DnB NOR Bank</td>
<td>GETCO Ltd.</td>
</tr>
<tr>
<td>Deutsche Bank (London Branch)</td>
<td>Handelsbanken</td>
</tr>
<tr>
<td>HSBC Trinkaus &amp; Burkhardt</td>
<td>Jefferies International Ltd.</td>
</tr>
<tr>
<td>Istituto Centrale delle Banche Popolari Italiane SpA</td>
<td>Knight Capital Markets</td>
</tr>
<tr>
<td>Interactive Brokers</td>
<td>Lan &amp; Spar Bank A/S</td>
</tr>
<tr>
<td>KBC Bank N.V.</td>
<td>Nordnet Bank AB</td>
</tr>
<tr>
<td>Nordea</td>
<td>Nomura International Plc</td>
</tr>
<tr>
<td>Swedbank</td>
<td>Nykredit A/S</td>
</tr>
<tr>
<td>Credit Agricole Cheuvreux</td>
<td>Pohjola Bank</td>
</tr>
<tr>
<td>Credit Suisse Securities (europe) Ltd</td>
<td>RBC Capital Markets</td>
</tr>
<tr>
<td>Morgan Stanley International Plc</td>
<td>Saxo Bank A/S</td>
</tr>
<tr>
<td>RBS Bank N.V.</td>
<td>Spar Nord Bank A/S</td>
</tr>
<tr>
<td>Instinet europe Ltd.</td>
<td>Sparekassens Kronjylland A/S</td>
</tr>
<tr>
<td>Morgan Stanley Securities Ltd.</td>
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</table>

Source: Zhu (2011)
## Summary statistics

<table>
<thead>
<tr>
<th>Panel A: Overall summary statistics</th>
<th>Mean</th>
<th>Std</th>
<th>Min</th>
<th>Median</th>
<th>Max</th>
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</thead>
<tbody>
<tr>
<td>Daily number of reports</td>
<td>6,293.6</td>
<td>699.0</td>
<td>1,135.0</td>
<td>6,426.5</td>
<td>7,663.0</td>
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<tr>
<td>Daily volume (in mln shares)</td>
<td>160.9</td>
<td>42.1</td>
<td>8.1</td>
<td>155.5</td>
<td>342.4</td>
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<tr>
<td>Daily volume (in mln euro)</td>
<td>1,809.8</td>
<td>475.1</td>
<td>272.4</td>
<td>1,762.3</td>
<td>3,649.6</td>
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<tr>
<td>Volume per report (in 1000 shares)</td>
<td>25.6</td>
<td>114.1</td>
<td>0.0</td>
<td>2.6</td>
<td>18,631.8</td>
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<tr>
<td>Volume per report (in 1000 euro)</td>
<td>287.6</td>
<td>1,067.6</td>
<td>0.0</td>
<td>36.1</td>
<td>142,271.3</td>
</tr>
</tbody>
</table>

### Panel B: Cross-sectional summary statistics, based on clearing-member averages

| Daily number of reports            | 114.4  | 143.7 | 0.0   | 64.9   | 736.4 |
| Daily volume (in mln shares)       | 2.9    | 4.2   | 0.0   | 0.7    | 20.8 |
| Daily volume (in mln euro)         | 32.9   | 46.9  | 0.0   | 7.8    | 222.4 |

### Panel C: Cross-sectional summary statistics, based on stock averages

| Daily number of reports            | 26.0   | 21.9  | 0.0   | 20.6   | 84.2 |
| Daily volume (in mln shares)       | 0.7    | 1.6   | 0.0   | 0.1    | 14.2 |
| Daily volume (in mln euro)         | 7.5    | 14.6  | 0.0   | 0.9    | 124.0 |
Margin collected vs. ExpCCP

Apr 22: Nokia publishes Q1 results
May 9: EU announces bailout program

MarginCCP

Apr 22:
Nokia publishes Q1 results
May 9:
EU announces bailout program

Marginal collected vs. ExpCCP

0
100
200
300
400
500
600
700
800

0
100
200
300
400
500
600
700
800

million euro

Nov 2009
Dec 2009
Jan 2010
Feb 2010
Mar 2010
Apr 2010
May 2010
Jun 2010
Jul 2010
Aug 2010
Sep 2010
Margin collected vs. ExpCCP

Apr 22: Nokia publishes Q1 results
May 9: EU announces bailout program

ExpCCP
Margin

(million euro)
Margin collected vs. ExpCCP

Apr 21: Nokia publishes Q1 results
May 2: Eurozone and IMF agree to bailout Greece
May 9: EU announces bailout program

Margin collected

ExpCCP

CrowdIx (right)
Aggregate loss distribution Nokia day

![Graph showing the distribution of aggregate losses in million euros for Nokia's Q1 reports compared to the Median and Min CrowdIx day benchmarks.](image-url)
Margin collected versus $\text{ExpCCP}$

$\text{CrowdIx} = 0.72$

The chart depicts the relationship between the margin posted by member $j$, $\text{Margin}_j$ (in million euros), and the member's contribution to CCP exposure, $\text{ExpCCP}_j$ (in million euros). The formula for $\text{CrowdIx}$ is shown in the image as 0.72.
### Margin collected versus ExpCCP

<table>
<thead>
<tr>
<th>Stock</th>
<th>NetPos (mln €)</th>
<th>AbsNetPos (mln €)</th>
<th>AbsNetPos (%)</th>
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</thead>
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<tr>
<td>NOKI</td>
<td>-84.7</td>
<td>84.7</td>
<td>20.7</td>
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<tr>
<td>ER</td>
<td>64.8</td>
<td>64.8</td>
<td>15.8</td>
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<tr>
<td>FUM1V</td>
<td>-39.2</td>
<td>39.2</td>
<td>9.6</td>
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<td>NDA1V</td>
<td>-31.7</td>
<td>31.7</td>
<td>7.7</td>
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<tr>
<td>VOLB</td>
<td>16.2</td>
<td>16.2</td>
<td>4.0</td>
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<tr>
<td>HMB</td>
<td>15.5</td>
<td>15.5</td>
<td>3.8</td>
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<tr>
<td>STERV</td>
<td>15.3</td>
<td>15.3</td>
<td>3.7</td>
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<tr>
<td>TLS1V</td>
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<td>9.8</td>
<td>2.4</td>
</tr>
<tr>
<td>OUT1V</td>
<td>-8.9</td>
<td>8.9</td>
<td>2.2</td>
</tr>
<tr>
<td>SEN</td>
<td>-8.3</td>
<td>8.3</td>
<td>2.0</td>
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<th>Clearing member 12</th>
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<tr>
<td>Stock</td>
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<td>VOLB</td>
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<td>TLS1V</td>
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<td>TRELB</td>
</tr>
<tr>
<td>TEL2B</td>
</tr>
<tr>
<td>ASSAB</td>
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<tr>
<td>BOLI</td>
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</table>
Dispersion across members gap \( ExpCCP \)-margin

![Graph showing the dispersion of \( ExpCCP \)-margin across different dates with dates such as May 10, 2010, April 23, 2010, etc., and values ranging from -100 to +150 million euros.]
**Correlation of this gap across days**

<table>
<thead>
<tr>
<th></th>
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<tbody>
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<td>1.00</td>
<td>0.36*</td>
<td>0.35*</td>
<td>0.63*</td>
<td>0.73*</td>
<td>0.03</td>
<td>0.06</td>
<td>0.77*</td>
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<tr>
<td>Apr 23, 2010</td>
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<td>0.71*</td>
<td>0.41*</td>
<td>0.35*</td>
<td>0.08</td>
<td>0.16</td>
<td>0.31*</td>
<td>0.23</td>
<td>0.08</td>
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<tr>
<td>Apr 26, 2010</td>
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<td>0.24</td>
<td>0.62*</td>
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<td>0.40*</td>
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<td>0.59*</td>
<td>−0.06</td>
<td>0.11</td>
<td>0.74*</td>
<td>0.19</td>
<td>0.05</td>
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<tr>
<td>May 11, 2010</td>
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<td>−0.16</td>
<td>0.62*</td>
<td>0.07</td>
<td>0.21</td>
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<td>Feb 01, 2010</td>
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<td>−0.24</td>
<td>0.29*</td>
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<td>Aug 20, 2010</td>
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<td>0.18</td>
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<td>May 21, 2010</td>
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<td>0.00</td>
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<td>May 14, 2010</td>
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<td>0.25</td>
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<td></td>
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</tr>
<tr>
<td>Jun 11, 2010</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1.00</td>
<td></td>
</tr>
</tbody>
</table>

*: Significant at the 5% level.
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Conclusion

1. Crowded trades constitute a hidden risk to a CCP.
Conclusion

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2. *CrowdIx* proposed as thermometer for crowded-trade risk.
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   3.5 It extrapolates standard practice which should make introduction easier.
4. The implementation on real data shows that it matters, in particular when the market gets turbulent.
Crowded Risk as a Systemic Concern for Central Clearing Counterparties

Albert J. Menkveld

VU Amsterdam and Tinbergen Institute

November 1, 2016
Outline

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Objective

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Appendix

References
Appendix A: Max crowding benchmark, $\tilde{A}$

1. If all members would trade the same risk factor, then $\exists n \in \mathbb{R}^I$ s.t. $\forall j$:

   $$X_j = \nu_j \times (n' R), \quad \nu_j \in \mathbb{R}.$$ 

2. Then,

   $$\Sigma = n' \Omega n \times \left( \nu_j \nu'_j \right).$$

3. Without loss of generality, let $n' \Omega n = 1$.

4. For member by member portfolio risks to remain unchanged, one needs $\forall j$:

   $$\nu_j^2 = \sigma_j^2 \quad \Rightarrow \quad \nu_j = \pm \sqrt{\sigma_j^2}. \quad (1)$$

5. In addition, the aggregate (signed) trade is zero:

   $$\sum_j \nu_j = 0. \quad (2)$$
Appendix A: Max crowding benchmark, $\tilde{\mathbf{A}}$

1. The member trade reallocation that yields the maximum crowding benchmark is

$$\text{argmax} \min \left( \sum_j \nu_j^+, \sum_j \nu_j^- \right)$$

subject to (1),

$$\text{subject to (1),}$$

where

$$\nu_j^+ := \max (\nu_j, 0) \text{ and } \nu_j^- := \max (-\nu_j, 0).$$

2. If $\sum_j \nu_j^+ = \sum_j \nu_j^+$ then trade reallocation is perfect. No portfolio risk is left unallocated.
Appendix A: Max crowding benchmark, $\tilde{A}$

1. The trade reallocation is a combinatorial problem that is NP hard.

2. It maps into a one-dimensional bin packing problem (Coffman, Garey, and Johnson, 1996). Can all items be packed into two bins of size $(1/2) \sum_j \sigma_j^2$? If not, how much can be packed into two such bins? The minimum of the two bins can be matched, i.e., buyers buy this amount from sellers.

3. First fit descending (FFD) algorithm solves the offline bin packing problem in $O(J \log J)$ time (brute force requires $3^J$).

4. Why FFD instead of alternative approaches?
   4.1 Average-case analysis: If item size is drawn from $U[0, 1/2]$ for one-unit bins then Coffman, Garey, and Johnson (1996, p. 39) claim “FFD is typically optimal.”
   4.2 Worst-case analysis: If all items are smaller than 1/2 then FFD does as well its closest contender MFFD (modified first fit descending) (Coffman, Garey, and Johnson, 1996, p. 16-19).
Appendix B: Q&A

1. **Is it reasonable to assume equity returns are normal?** In the implementation the return distribution is assumed to be *conditionally* normal. Time-varying volatility is accounted for by calculating the covariance matrix as an exponentially weighted average of the outer product of historical daily returns.\(^2\)

\(^2\)EWMA(0.94) which is the RiskMetrics standard for daily equity returns.


Hedegaard, Esben. 2012. “How Margins are Set and Affect Prices.” Manuscript, NYU.


