



Crowded Risk as a Systemic Concern for Central Clearing Counterparties

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 - 2.1 Structural reforms . . . **improved risk management throughout the financial system**. In particular, the mandatory move to clearing standardised over-the-counter (OTC) derivatives trades via CCPs will help to reduce counterparty risk between financial institutions . . .

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1. Bernanke (2011) emphasized financial stability strongly depends on resiliency of CCP.
2. ESRB annual report 2012, p. 16:
 - 2.1 Structural reforms . . . **improved risk management throughout the financial system**. In particular, the mandatory move to clearing standardised over-the-counter (OTC) derivatives trades via CCPs will help to reduce counterparty risk between financial institutions . . .
 - 2.2 However, the more prominent role of CCPs will also introduce new systemic risks. Mandatory clearing will turn **CCPs into systemic nodes** in the financial system, with unknown, but possibly far-reaching, consequences.

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 - 1.2 BIS-IOSCO (2012)
“Principles for Financial Market Infrastructures.”

Exhibit 1:
CPSS-IOSCO Technical Committee
Recommendations for Central Counterparties (CCPs)

1. Legal risk

A CCP should have a well founded, transparent and enforceable legal framework for each aspect of its activities in all relevant jurisdictions.

2. Participation requirements

A CCP should require participants to have sufficient financial resources and robust operational capacity to meet obligations arising from participation in the CCP. A CCP should have procedures in place to monitor that participation requirements are met on an ongoing basis. A CCP's participation requirements should be objective, publicly disclosed, and permit fair and open access.

3. Measurement and management of credit exposures

A CCP should measure its credit exposures to its participants at least once a day. Through margin requirements, other risk control mechanisms or a combination of both, a CCP should limit its exposures to potential losses from defaults by its participants in normal market conditions so that the operations of the CCP would not be disrupted and non-defaulting participants would not be exposed to losses that they cannot anticipate or control.

4. Margin requirements

If a CCP relies on margin requirements to limit its credit exposures to participants, those requirements should be sufficient to cover potential exposures in normal market conditions. The models and parameters used in setting margin requirements should be risk-based and reviewed regularly.

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3. For example, 54 exchanges and clearing houses use SPAN developed by Chicago Mercantile Exchange (CME).

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Objective

1. Do crowded trades raise CCP exposure toward its members in a way not accounted for by member specific margins? **Yes!**
2. If so, can one come up with a reasonable measure of crowding? **Yes!**
3. And, is there a natural way to allocate CCP exposure across members (according to the “polluter pays” principle)? **Yes!**

Findings

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 - 3.1 *CrowdIx*: A crowding index to measure crowded-trade risk.
 - 3.2 *ExpCCP*: A CCP exposure measure with decomposition across members (polluter pays).

Literature

1. CCP literature

- 1.1 **CCP introduction:** Duffie and Zhu (2011), Koepl, Monnet, and Temzelides (2012), Menkveld, Pagnotta, and Zoican (2015).
- 1.2 **CCP monitoring incentives:** Biais, Heider, and Hoerova (2011), Acharya and Bisin (2011), Koepl (2013).
- 1.3 **CCP risk management:** Cruz Lopez et al. (2016), Hedegaard (2012), Jones and Pérignon (2013), Menkveld (2015), Capponi, Cheng, and Rajan (2016).

2. Systemic risk literature

- 2.1 **Crowded trades:** Khandani and Lo (2007), Khandani and Lo (2011), Pojarliev and Levich (2011).
- 2.2 **Systemic risk in trades:** Basak and Shapiro (2001), Acharya (2009), Farhi and Tirole (2012).
- 2.3 **Systemic risk measurement/allocation:** Bisias et al. (2012), Brunnermeier and Cheridito (2014), Capponi (2016).

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CCP exposure measure in Duffie-Zhu

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2. n_j is the vector of yet-to-settle trade portfolio of member j .
3. Let $X_j = n_j' R$ be the P&L on member j 's trade portfolio, then

$$X \sim N(0, \Sigma), \quad \Sigma = N' \Omega N, \quad N = [n_1, \dots, n_J].$$

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$$X \sim N(0, \Sigma), \quad \Sigma = N' \Omega N, \quad N = [n_1, \dots, n_J].$$

4. CCP exposure to trade portfolios of all members is defined as

$$E(A) = E\left(\sum_j E_j\right) \quad \text{with } E_j = -\min(X_j, 0).$$

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Definition

Let ExpCCP be the total margin a CCP should collect to protect against tail risk:

$$\text{ExpCCP} := E(A) + \alpha \text{std}(A).$$

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Definition

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$$\text{ExpCCP} := E(A) + \alpha \text{std}(A).$$

$E(A) = \sum_j \sqrt{\frac{1}{2\pi}} \sigma_j$ (Duffie and Zhu, 2011), but what about $\text{std}(A)$?

Absolute Moments in 2-dimensional Normal Distribution

By Seiji NABEYA

Let x and y be distributed according to the following 2-dimensional normal distribution,

$$\frac{1}{2\pi\sigma_1\sigma_2\sqrt{1-\rho^2}} \exp \left\{ -\frac{1}{2(1-\rho^2)} \left(\frac{x^2}{\sigma_1^2} - \frac{xy}{\sigma_1\sigma_2} + \frac{y^2}{\sigma_2^2} \right) \right\} dx dy.$$

It is our purpose to express absolute moments in terms of elementary functions. Putting $E(|x^m y^n|) = (m, n)$ for simplicity, we have

$$\begin{aligned} (m, n) &= \frac{1}{2\pi\sigma_1\sigma_2\sqrt{1-\rho^2}} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} |x^m y^n| \exp \left\{ -\frac{1}{2(1-\rho^2)} \left(\frac{x^2}{\sigma_1^2} - 2\rho \frac{xy}{\sigma_1\sigma_2} + \frac{y^2}{\sigma_2^2} \right) \right\} dx dy \\ &= \frac{2^{\frac{m+n}{2}} \sigma_1^m \sigma_2^n}{\pi} (1-\rho^2)^{\frac{m+n+1}{2}} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} |x^m y^n| \exp(-x^2 + 2\rho xy - y^2) dx dy \\ &= \frac{2^{\frac{m+n}{2}} \sigma_1^m \sigma_2^n}{\pi} (1-\rho^2)^{\frac{m+n+1}{2}} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} |x^m y^n| e^{-x^2-y^2} \sum_{k=0}^{\infty} \frac{(2\rho xy)^k}{k!} dx dy \\ &= \frac{2^{\frac{m+n}{2}} \sigma_1^m \sigma_2^n}{\pi} (1-\rho^2)^{\frac{m+n+1}{2}} \sum_{k=0}^{\infty} \frac{\Gamma\left(\frac{m+1}{2} + k\right) \Gamma\left(\frac{n+1}{2} + k\right)}{(2k)!} (2\rho)^k \\ &= \frac{2^{\frac{m+n}{2}} \sigma_1^m \sigma_2^n}{\pi} \Gamma\left(\frac{m+1}{2}\right) \Gamma\left(\frac{n+1}{2}\right) (1-\rho^2)^{\frac{m+n+1}{2}} \\ &\quad \times F\left(\frac{m+1}{2}, \frac{n+1}{2}; \frac{1}{2}; \rho^2\right) \\ &= \frac{2^{\frac{m+n}{2}} \sigma_1^m \sigma_2^n}{\pi} \Gamma\left(\frac{m+1}{2}\right) \Gamma\left(\frac{n+1}{2}\right) F\left(-\frac{m}{2}, -\frac{n}{2}; \frac{1}{2}; \rho^2\right). \end{aligned}$$

Here

$$F(\alpha, \beta; \gamma; z) = 1 + \frac{\alpha\beta}{1!\gamma} z + \frac{\alpha(\alpha+1)\beta(\beta+1)}{2!\gamma(\gamma+1)} z^2 + \dots$$

is the hypergeometric function, which reduces to the polynomial of z if α or β is a non-positive integer and γ is positive. Thus, when at least one of the integers m, n is an even number, (m, n) reduces to the polynomial of ρ^2 multiplied by $\sigma_1^m \sigma_2^n$.

The case where both m and n are odd may be treated as follows. Put

$$x = \sqrt{2(1-\rho^2)} \sigma_1 r \cos \theta, \quad y = \sqrt{2(1-\rho^2)} \sigma_2 r \sin \theta.$$

When $m - n = 2q$, where q is a non-negative integer, we have then

$$\begin{aligned} (m, n) &= \frac{1}{2\pi\sigma_1\sigma_2\sqrt{1-\rho^2}} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} |x^m y^n| \exp\left\{-\frac{1}{2(1-\rho^2)}\right. \\ &\quad \times \left(\frac{x^2}{\sigma_1^2} - 2\rho \frac{xy}{\sigma_1\sigma_2} + \frac{y^2}{\sigma_2^2}\right)\bigg\} dx dy \\ &= \frac{2^{\frac{m+n}{2}} \sigma_1^m \sigma_2^n}{\pi} (1-\rho^2)^{\frac{m+n+1}{2}} \int_0^{2\pi} \int_0^{\infty} r^{m+n+1} |\cos^m \theta \sin^n \theta| \\ &\quad \times \exp\{-r^2(1-2\rho \cos \theta \sin \theta)\} dr d\theta \\ &= \frac{2^{\frac{m+n-2}{2}} \sigma_1^m \sigma_2^n}{\pi} \Gamma\left(\frac{m+n}{2} + 1\right) (1-\rho^2)^{\frac{m+n+1}{2}} \\ &\quad \times \int_0^{2\pi} \frac{|\cos^m \theta \sin^n \theta|}{(1-2\rho \cos \theta \sin \theta)^{\frac{m+n+2}{2}}} d\theta \\ &= \frac{2^{\frac{m+n}{2}} \sigma_1^m \sigma_2^n}{\pi} \Gamma\left(\frac{m+n}{2} + 1\right) (1-\rho^2)^{\frac{m+n+1}{2}} \\ &\quad \times \int_0^{\frac{\pi}{2}} \left\{ \frac{\cos^m \theta \sin^n \theta}{(1-2\rho \cos \theta \sin \theta)^{\frac{m+n+2}{2}}} + \frac{\cos^m \theta \sin^n \theta}{(1+2\rho \cos \theta \sin \theta)^{\frac{m+n+2}{2}}} \right\} d\theta \\ &= \frac{2^{\frac{m-n}{2}} \sigma_1^m \sigma_2^n}{\pi} \Gamma\left(\frac{m-n}{2} + 1\right) (1-\rho^2)^{\frac{m+n+1}{2}} \\ &\quad \times \frac{d^n}{d\rho^n} \int_0^{\frac{\pi}{2}} \left\{ \frac{\cos^{2n} \theta}{(1-2\rho \cos \theta \sin \theta)^{n+1}} - \frac{\cos^{2n} \theta}{(1+2\rho \cos \theta \sin \theta)^{n+1}} \right\} d\theta. \end{aligned}$$

As the last integral may be calculated in the elementary fashion, (m, n) may be evaluated.

In the following we shall give the obtained formulae for the cases $m \geq n$. The formula of (m, n) for $m \leq n$, is obtained by exchanging σ_1 and σ_2 in the formula (n, m) .

$$(1, 0) = \sqrt{\frac{2}{\pi}} \sigma_1,$$

$$(2, 0) = \sigma_1^2,$$

$$(1, 1) = \frac{2}{\pi} \left(\sqrt{1-\rho^2} + \rho \sin^{-1} \rho \right) \sigma_1 \sigma_2,$$

$$(3, 0) = 2\sqrt{\frac{2}{\pi}} \sigma_1^3,$$

$$(2, 1) = \sqrt{\frac{2}{\pi}} (1 + \rho^2) \sigma_1^2 \sigma_2,$$

$$(4, 0) = 3\sigma_1^4,$$

$$(3, 1) = \frac{2}{\pi} \left\{ \sqrt{1 - \rho^2} (2 + \rho^2) + 3\rho \sin^{-1} \rho \right\} \sigma_1^3 \sigma_2,$$

$$(2, 2) = (1 + 2\rho^2) \sigma_1^2 \sigma_2^2,$$

$$(5, 0) = 8\sqrt{\frac{2}{\pi}} \sigma_1^5,$$

$$(4, 1) = \sqrt{\frac{2}{\pi}} (3 + 6\rho^2 - \rho^4) \sigma_1^4 \sigma_2,$$

$$(3, 2) = 2\sqrt{\frac{2}{\pi}} (1 + 3\rho^2) \sigma_1^3 \sigma_2^2,$$

$$(6, 0) = 15\sigma_1^6,$$

$$(5, 1) = \frac{2}{\pi} \left\{ \sqrt{1 - \rho^2} (8 + 9\rho^2 - 2\rho^4) + 15\rho \sin^{-1} \rho \right\} \sigma_1^5 \sigma_2,$$

$$(4, 2) = 3(1 + 4\rho^2) \sigma_1^4 \sigma_2^2,$$

$$(3, 3) = \frac{2}{\pi} \left\{ \sqrt{1 - \rho^2} (4 + 11\rho^2) + 3\rho (3 + 2\rho^2) \sin^{-1} \rho \right\} \sigma_1^3 \sigma_2^3,$$

$$(7, 0) = 48\sqrt{\frac{2}{\pi}} \sigma_1^7,$$

$$(6, 1) = 3\sqrt{\frac{2}{\pi}} (5 + 15\rho^2 - 5\rho^4 + \rho^6) \sigma_1^6 \sigma_2,$$

$$(5, 2) = 8\sqrt{\frac{2}{\pi}} (1 + 5\rho^2) \sigma_1^5 \sigma_2^2,$$

$$(4, 3) = 6\sqrt{\frac{2}{\pi}} (1 + 6\rho^2 + \rho^4) \sigma_1^4 \sigma_2^3,$$

$$(8, 0) = 105\sigma_1^8,$$

$$(7, 1) = \frac{2}{\pi} \left\{ \sqrt{1 - \rho^2} (48 + 87\rho^2 - 38\rho^4 + 8\rho^6) + 105\rho \sin^{-1} \rho \right\} \sigma_1^7 \sigma_2,$$

$$(6, 2) = 15(1 + 6\rho^2) \sigma_1^6 \sigma_2^2,$$

$$(5, 3) = \frac{2}{\pi} \left\{ \sqrt{1 - \rho^2} (16 + 83\rho^2 + 6\rho^4) + 15\rho (3 + 4\rho^2) \sin^{-1} \rho \right\} \sigma_1^5 \sigma_2^3,$$

$$(4, 4) = 3(3 + 24\rho^2 + 8\rho^4) \sigma_1^4 \sigma_2^4,$$

$$(9, 0) = 384 \sqrt{\frac{2}{\pi}} \sigma_1^9,$$

$$(8, 1) = 3 \sqrt{\frac{2}{\pi}} (35 + 140\rho^2 - 70\rho^4 + 28\rho^6 - 5\rho^8) \sigma_1^8 \sigma_2,$$

$$(7, 2) = 48 \sqrt{\frac{2}{\pi}} (1 + 7\rho^2) \sigma_1^7 \sigma_2^2,$$

$$(6, 3) = 6 \sqrt{\frac{2}{\pi}} (5 + 45\rho^2 + 15\rho^4 - \rho^6) \sigma_1^6 \sigma_2^3,$$

$$(5, 4) = 24 \sqrt{\frac{2}{\pi}} (1 + 10\rho^2 + 5\rho^4) \sigma_1^5 \sigma_2^4,$$

$$(10, 0) = 945 \sigma_1^{10},$$

$$(9, 1) = \frac{6}{\pi} \left\{ \sqrt{1 - \rho^2} (128 + 325\rho^2 - 210\rho^4 + 88\rho^6 - 16\rho^8) \right. \\ \left. + 315\rho \sin^{-1}\rho \right\} \sigma_1^9 \sigma_2,$$

$$(8, 2) = 105 (1 + 8\rho^2) \sigma_1^8 \sigma_2^2,$$

$$(7, 3) = \frac{2}{\pi} \left\{ \sqrt{1 - \rho^2} (96 + 741\rho^2 + 120\rho^4 - 12\rho^6) \right. \\ \left. + 315\rho (1 + 2\rho^2) \sin^{-1}\rho \right\} \sigma_1^7 \sigma_2^3,$$

$$(6, 4) = 45 (1 + 12\rho^2 + 8\rho^4) \sigma_1^6 \sigma_2^4,$$

$$(5, 5) = \frac{2}{\pi} \left\{ \sqrt{1 - \rho^2} (64 + 607\rho^2 + 274\rho^4) \right. \\ \left. + 15\rho (15 + 40\rho^2 + 8\rho^4) \sin^{-1}\rho \right\} \sigma_1^5 \sigma_2^5,$$

$$(11, 0) = 3840 \sqrt{\frac{2}{\pi}} \sigma_1^{11},$$

$$(10, 1) = 15 \sqrt{\frac{2}{\pi}} (63 + 315\rho^2 - 210\rho^4 + 126\rho^6 - 45\rho^8 + 7\rho^{10}) \sigma_1^{10} \sigma_2,$$

$$(9, 2) = 384 \sqrt{\frac{2}{\pi}} (1 + 9\rho^2) \sigma_1^9 \sigma_2^2,$$

$$(8, 3) = 6 \sqrt{\frac{2}{\pi}} (35 + 420\rho^2 + 210\rho^4 - 28\rho^6 + 3\rho^8) \sigma_1^8 \sigma_2^3,$$

$$(7, 4) = 48 \sqrt{\frac{2}{\pi}} (3 + 42\rho^2 + 35\rho^4) \sigma_1^7 \sigma_2^4,$$

$$(6, 5) = 120 \sqrt{\frac{2}{\pi}} (1 + 15\rho^2 + 15\rho^4 + \rho^6) \sigma_1^6 \sigma_2^5,$$

$$(12, 0) = 10395\sigma_1^{12},$$

$$(11, 1) = \frac{6}{\pi} \left\{ \sqrt{1-\rho^2} (1280 + 4215\rho^2 - 3590\rho^4 + 2248\rho^6 - 816\rho^8 \right. \\ \left. + 128\rho^{10}) + 3465\rho \sin^{-1}\rho \right\} \sigma_1^{11}\sigma_2,$$

$$(10, 2) = 945(1 + 10\rho^2)\sigma_1^{10}\sigma_2^2,$$

$$(9, 3) = \frac{6}{\pi} \left\{ \sqrt{1-\rho^2} (256 + 2639\rho^2 + 690\rho^4 - 136\rho^6 + 16\rho^8) \right. \\ \left. + 315\rho(3 + 8\rho^2) \sin^{-1}\rho \right\} \sigma_1^9\sigma_2^3,$$

$$(8, 4) = 315(1 + 16\rho^2 + 16\rho^4)\sigma_1^8\sigma_2^4,$$

$$(7, 5) = \frac{6}{\pi} \left\{ \sqrt{1-\rho^2} (128 + 1779\rho^2 + 1518\rho^4 + 40\rho^6) \right. \\ \left. + 105\rho(5 + 20\rho^2 + 8\rho^4) \sin^{-1}\rho \right\} \sigma_1^7\sigma_2^5,$$

$$(6, 6) = 45(5 + 90\rho^2 + 120\rho^4 + 16\rho^6)\sigma_1^6\sigma_2^6.$$

In another paper we shall treat the 3-dimensional case by a unified but more complicated method.

Institute of Statistical Mathematics.

CCP exposure

1. Some algebra using results for folded and truncated normal (Nabeya, 1951; Rosenbaum, 1961) yields:

$$\text{std}(A) = \sqrt{\sum_{k,l} \left(\frac{\pi - 1}{2\pi} \right) \sigma_k \sigma_l M(\rho_{kl})}$$

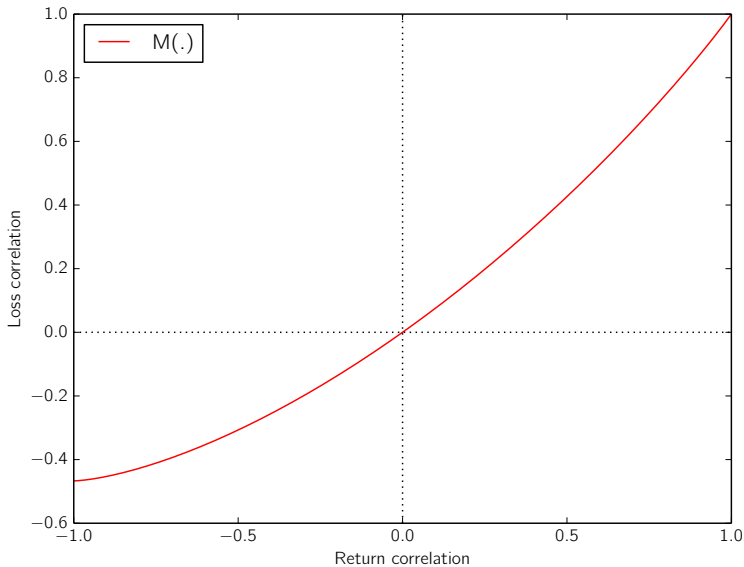
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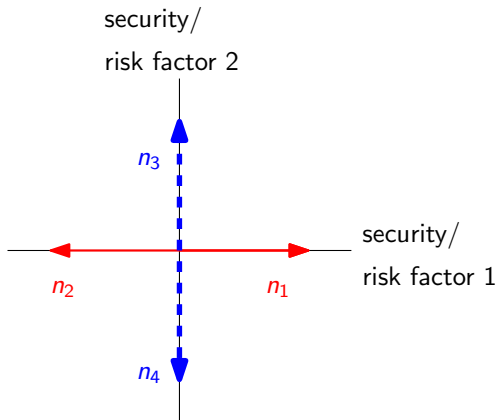
$$M(\rho) = \frac{\left[\frac{1}{2}\pi + \arcsin(\rho) \right] \rho + \sqrt{1-\rho^2} - 1}{\pi - 1}$$

CCP exposure



Simple example

Noncrowded trades



Simple example noncrowded trades

1.

$$N = \begin{pmatrix} 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 \end{pmatrix}, \Sigma = \begin{pmatrix} 1 & -1 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & -1 & 1 \end{pmatrix}$$

Simple example noncrowded trades

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2.

$$E(E) = \sqrt{\frac{1}{2\pi}} \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}, \text{var}(E) = \frac{1}{2\pi} \begin{pmatrix} \pi-1 & -1 & 0 & 0 \\ -1 & \pi-1 & 0 & 0 \\ 0 & 0 & \pi-1 & -1 \\ 0 & 0 & -1 & \pi-1 \end{pmatrix}$$

Simple example noncrowded trades

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$$N = \begin{pmatrix} 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 \end{pmatrix}, \Sigma = \begin{pmatrix} 1 & -1 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & -1 & 1 \end{pmatrix}$$

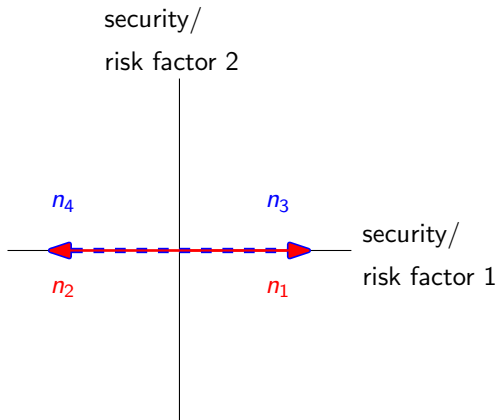
2.

$$E(E) = \sqrt{\frac{1}{2\pi}} \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}, \text{var}(E) = \frac{1}{2\pi} \begin{pmatrix} \pi-1 & -1 & 0 & 0 \\ -1 & \pi-1 & 0 & 0 \\ 0 & 0 & \pi-1 & -1 \\ 0 & 0 & -1 & \pi-1 \end{pmatrix}$$

3.

$$E(A) = 4\sqrt{\frac{1}{2\pi}} \approx 1.60 \quad \text{and} \quad \text{std}(A) = 2\sqrt{\frac{\pi-2}{2\pi}} \approx 0.85$$

Crowded trades



Simple example crowded trades

1.

$$N = \begin{pmatrix} 1 & -1 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \Sigma = N' \Omega N = \begin{pmatrix} 1 & -1 & 1 & -1 \\ -1 & 1 & -1 & 1 \\ 1 & -1 & 1 & -1 \\ -1 & 1 & -1 & 1 \end{pmatrix}$$

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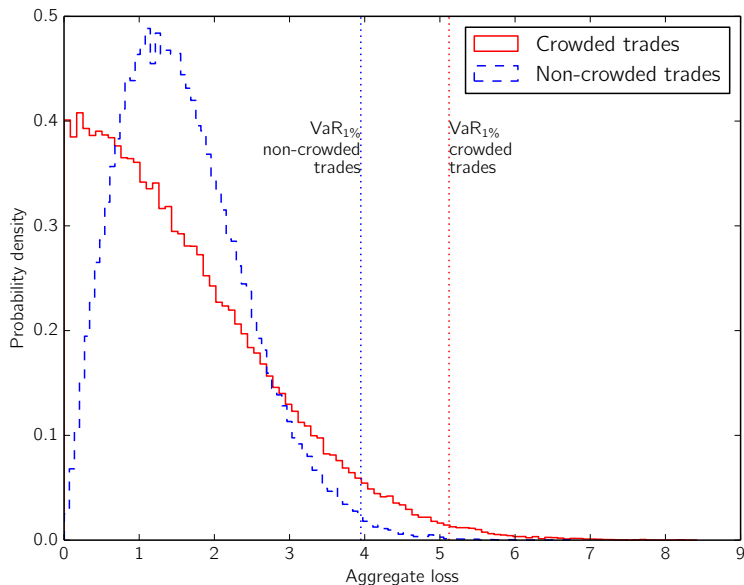
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3.

$$E(A) = 4 \sqrt{\frac{1}{2\pi}} \approx 1.60 \quad \text{and} \quad \text{std}(A) = 2 \sqrt{\frac{\pi-2}{\pi}} \approx 1.21$$

Histogram CCP exposure (N=4)



Two tools

Tool #1: *CrowdIx*

Is there a natural “thermometer” for crowded-trade risk?

¹A feasible approach to this NP hard problem is to convert it to a standard bin-packing problem which can be “solved” heuristically (see Appendix A of the slides).

Tool #1: *CrowdIx*

Is there a natural “thermometer” for crowded-trade risk?

Definition

CrowdIx for Σ is defined as

$$\text{CrowdIx} = \text{std}(A) / \text{std}(\tilde{A})$$

where \tilde{A} is CCP aggregate exposure when all members' trades are re-allocated to a single risk factor to the maximum extent possible.¹

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Lemma

$$\text{Crowdlx} \geq \sqrt{\frac{1}{\tilde{J}/2}} \quad \text{where } \tilde{J} = 2 \lfloor J/2 \rfloor J$$

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Tool #1: *CrowdIx*

1. CrowdIx in the simple example is

$$\begin{cases} \sqrt{1/2} = 0.71 & \text{in the noncrowded case.} \\ 1 & \text{in the crowded case.} \end{cases}$$

Tool #2: *ExpCCP*

1. Homogeneity of degree one of $E(A)$ and $\text{std}(A)$ implies that *ExpCCP* naturally decomposes across members (Euler's homogeneous function theorem).

1.1

$$E(A) = \sum_j \sqrt{\frac{1}{2\pi}} \sigma_j$$

1.2

$$\text{std}(A) = \sum_k \sigma_k \frac{\partial \text{std}(A)}{\partial \sigma_k} = \sum_k \sigma_k \sum_l \frac{1}{\text{std}(A)} \left(\frac{\pi-1}{2\pi} \right) \sigma_l M(\rho_{kl})$$

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- Therefore *ExpCCP* equals,

$$\sum_k \sigma_k \left(\underbrace{\sqrt{\frac{1}{2\pi}} + \frac{\alpha}{\text{std}(A)} \left(\frac{\pi-1}{2\pi} \right) \sigma_k}_{\text{Member-specific part ("old")}} + \underbrace{\sum_{l \neq k} \frac{\alpha}{\text{std}(A)} \left(\frac{\pi-1}{2\pi} \right) \sigma_l M(\rho_{kl})}_{\text{Crowded-trade part ("new")}} \right).$$

Tool #2: *ExpCCP*

1. To identify risk factor(s) on which members' trades crowd, the following results are useful:

1.1

$$\frac{\partial}{\partial \sigma^f} E(A) = \sum_j \sqrt{\frac{1}{2\pi}} \frac{\sigma_f}{\sigma_j} B_{jj}$$

1.2

$$\begin{aligned} \frac{\partial}{\partial \sigma^f} \text{std}(A) = & \left(\frac{\pi - 1}{4\pi} \right) \frac{\sigma^f}{\sigma^A} \sum_{k,l} \left[M'(\rho_{kl}) B_{kl} + \right. \\ & \left. + \frac{\rho_{kl}^2}{\pi - 1} \left(1 - 2\sqrt{1 - \rho_{kl}^2} \right) \left(\frac{\sigma_l}{\sigma_k} B_{kk} + \frac{\sigma_k}{\sigma_l} B_{ll} \right) \right] \end{aligned}$$

with

$$B_{kl} := n_k' \beta \beta' n_l \quad \text{and} \quad \beta = \text{cov}(R, r^f) / \text{var}(r^f)$$

Tool #2: *ExpCCP*

1. The sensitivity of *ExpCCP* to a particular risk factor is naturally described by the following elasticity:

$$e_{\sigma_f}^{ExpCCP} = \frac{\sigma_f}{ExpCCP} \left(\frac{\partial}{\partial \sigma^f} E(A) + \alpha \frac{\partial}{\partial \sigma^f} \text{std}(A) \right).$$

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Data

1. A European Multilateral Clearing Facility (EMCF) sample of trade reports filed by its (anonymized) members.

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5. Sample consists of 1.4 million trades by 57 clearing members in 242 securities across 228 days.

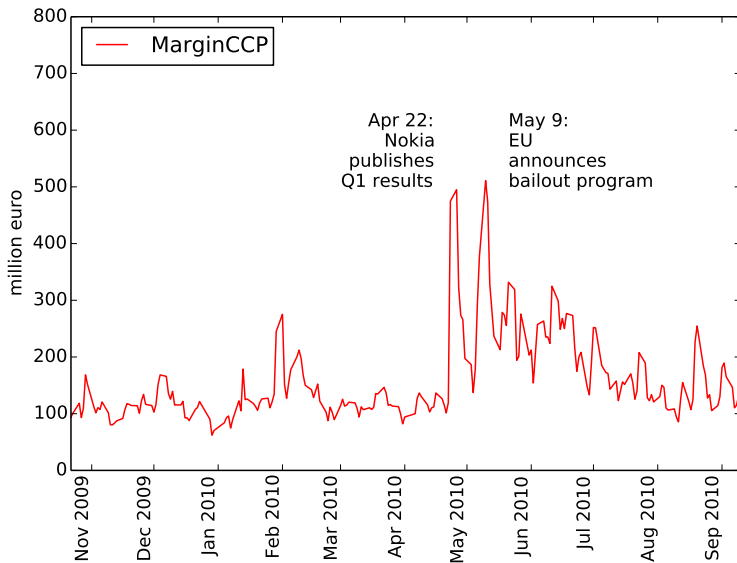
Clearing members

ABN AMRO Clearing Bank N.V.	Numis Securities Ltd
BNP Paribas Securities Services S.A.	UBS Ltd
Bank of America Merrill Lynch	Barclays Capital Securities Ltd.
Citibank Global Markets and Citibank International	Alandsbanken Abp
JPMorgan Securities Ltd.	Alandsbanken Sverige AB
Goldman Sachs International	Amagarbanken A/S
Skandinaviska Enskilda Banken	Arbejdernes Landsbank A/S
KAS BANK N.V.	Avanza Bank AB
Parel S.A.	Carnegie Bank A/S
Deutsche Bank AG	Dexia Securities France
Citigroup	E-Trade Bank
MF Global UK Ltd	Eik Bank A/S
CACEIS Bank Deutschland	EQ Bank Ltd.
Danske Bank	Evli Bank Plc
ABG Sundal Collier Norge	FIM Bank Ltd.
DnB NOR Bank	GETCO Ltd.
Deutsche Bank (London Branch)	Handelsbanken
HSBC Trinkaus & Burkhardt	Jefferies International Ltd.
Istituto Centrale delle Banche Popolari Italiane SpA	Knight Capital Markets
Interactive Brokers	Lan & Spar Bank A/S
KBC Bank N.V.	Nordnet Bank AB
Nordea	Nomura International Plc
Swedbank	Nykredit A/S
Credit Agricole Cheuvreux	Pohjola Bank
Credit Suisse Securities (europe) Ltd	RBC Capital Markets
Morgan Stanley International Plc	Saxo Bank A/S
RBS Bank N.V.	Spar Nord Bank A/S
Instinet europe Ltd.	Sparekassen Kronjylland A/S
Morgan Stanley Securities Ltd.	

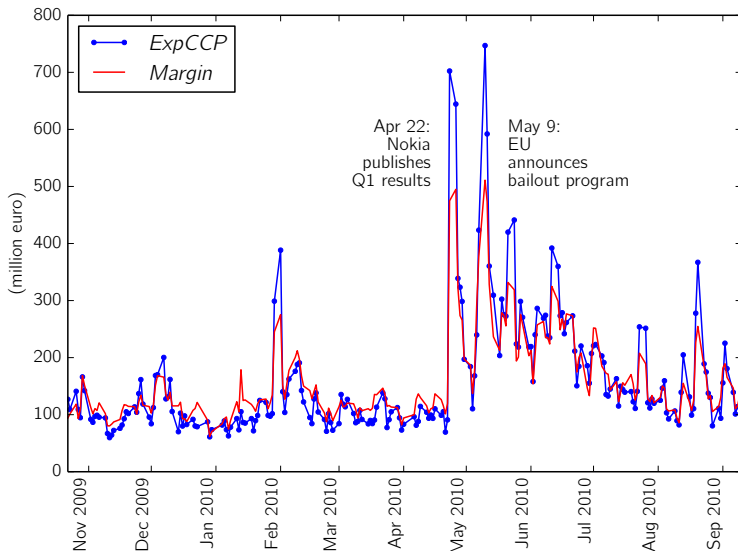
Summary statistics

	Mean	Std	Min	Median	Max
<i>Panel A: Overall summary statistics</i>					
Daily number of reports	6,293.6	699.0	1,135.0	6,426.5	7,663.0
Daily volume (in mln shares)	160.9	42.1	8.1	155.5	342.4
Daily volume (in mln euro)	1,809.8	475.1	272.4	1,762.3	3,649.6
Volume per report (in 1000 shares)	25.6	114.1	0.0	2.6	18,631.8
Volume per report (in 1000 euro)	287.6	1,067.6	0.0	36.1	142,271.3
<i>Panel B: Cross-sectional summary statistics, based on clearing-member averages</i>					
Daily number of reports	114.4	143.7	0.0	64.9	736.4
Daily volume (in mln shares)	2.9	4.2	0.0	0.7	20.8
Daily volume (in mln euro)	32.9	46.9	0.0	7.8	222.4
<i>Panel C: Cross-sectional summary statistics, based on stock averages</i>					
Daily number of reports	26.0	21.9	0.0	20.6	84.2
Daily volume (in mln shares)	0.7	1.6	0.0	0.1	14.2
Daily volume (in mln euro)	7.5	14.6	0.0	0.9	124.0

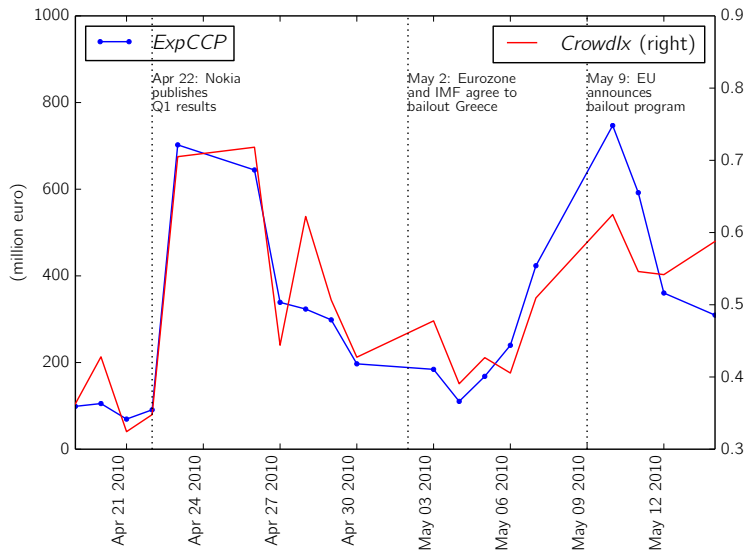
Margin collected vs. *ExpCCP*



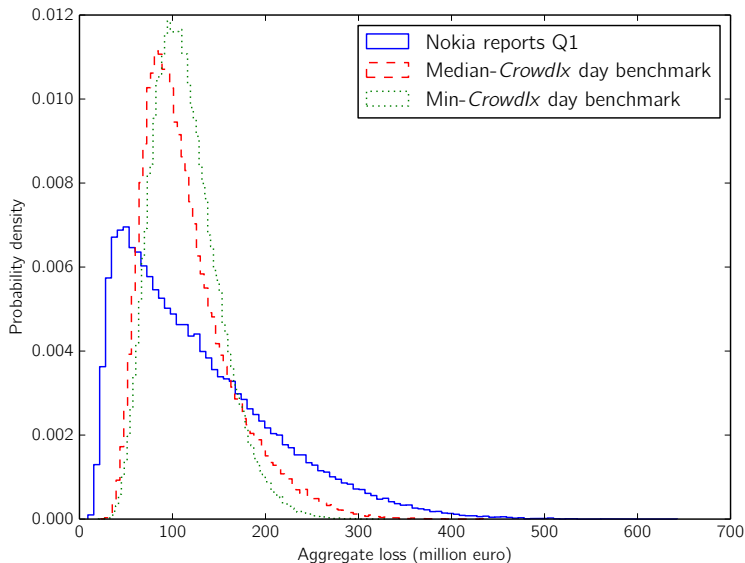
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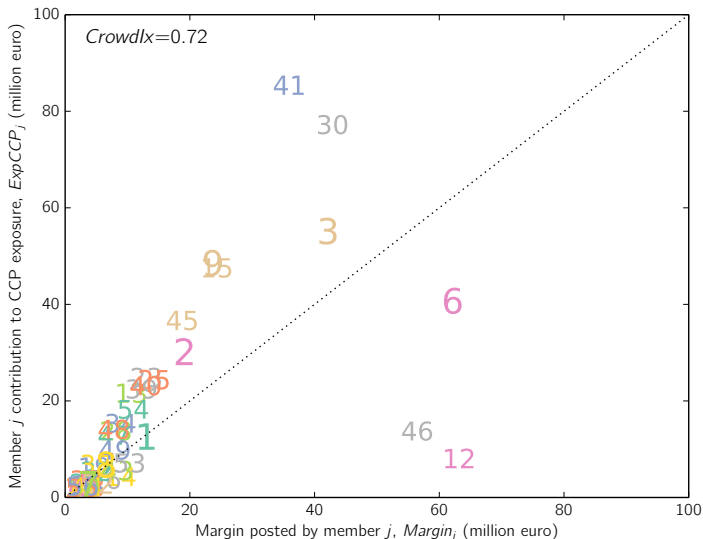
Margin collected vs. *ExpCCP*



Aggregate loss distribution Nokia day



Margin collected versus *ExpCCP*



Margin collected versus *ExpCCP*

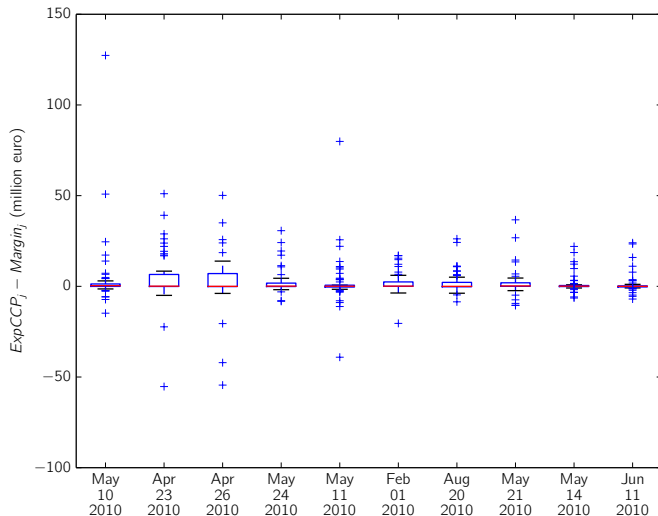
Clearing member 41

Stock	NetPos (mln €)	AbsNetPos (mln €)	AbsNetPos (%)
NOKI	-84.7	84.7	20.7
ER	64.8	64.8	15.8
FUM1V	-39.2	39.2	9.6
NDA1V	-31.7	31.7	7.7
VOLB	16.2	16.2	4.0
HMB	15.5	15.5	3.8
STERV	15.3	15.3	3.7
TLS1V	9.8	9.8	2.4
OUT1V	-8.9	8.9	2.2
SEN	-8.3	8.3	2.0

Clearing member 12

Stock	NetPos (mln €)	AbsNetPos (mln €)	AbsNetPos (%)
VOLB	35.7	35.7	12.6
TLS1V	-17.4	17.4	6.2
MAERS	-15.2	15.2	5.4
ABBN	-13.2	13.2	4.7
ALFA	-9.7	9.7	3.4
VWS	-9.2	9.2	3.2
TRELB	-9.0	9.0	3.2
TEL2B	-8.7	8.7	3.1
ASSAB	6.8	6.8	2.4
BOLI	6.3	6.3	2.2

Dispersion across members gap *ExpCCP*-margin



Correlation of this gap across days

	May 10, 2010	Apr 23, 2010	Apr 26, 2010	May 24, 2010	May 11, 2010	Feb 01, 2010	Aug 20, 2010	May 21, 2010	May 14, 2010	Jun 11, 2010
May 10, 2010	1.00	0.36*	0.35*	0.63*	0.73*	0.03	0.06	0.77*	0.13	0.18
Apr 23, 2010		1.00	0.71*	0.41*	0.35*	0.08	0.16	0.31*	0.23	0.08
Apr 26, 2010			1.00	0.24	0.62*	0.42*	0.13	0.40*	-0.20	0.05
May 24, 2010				1.00	0.59*	-0.06	0.11	0.74*	0.19	0.05
May 11, 2010					1.00	0.19	-0.16	0.62*	0.07	0.21
Feb 01, 2010						1.00	0.18	0.16	-0.24	0.29*
Aug 20, 2010							1.00	0.18	-0.05	0.18
May 21, 2010								1.00	-0.07	0.00
May 14, 2010									1.00	0.25
Jun 11, 2010										1.00

*: Significant at the 5% level.

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 - 3.5 It extrapolates standard practice which should make introduction easier.
4. The implementation on real data shows that it **matters**, in particular when the market gets turbulent.

Crowded Risk as a Systemic Concern for Central Clearing Counterparties

Albert J. Menkveld

VU Amsterdam and Tinbergen Institute

November 1, 2016

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Appendix A: Max crowding benchmark, \tilde{A}

1. If all members would trade the same risk factor, then $\exists n \in \mathbb{R}^I$ s.t. $\forall j$:

$$X_j = v_j \times (n'R), \quad v_j \in \mathbb{R}.$$

2. Then,

$$\Sigma = \underset{1 \times 1}{n' \Omega n} \times \underset{J \times J}{(v_j v_j')}.$$

3. Without loss of generality, let $n' \Omega n = 1$.
4. For member by member portfolio risks to remain unchanged, one needs $\forall j$:

$$v_j^2 = \sigma_j^2 \quad \Rightarrow \quad v_j = \pm \sqrt{\sigma_j^2}. \quad (1)$$

5. In addition, the aggregate (signed) trade is zero:

$$\sum_j v_j = 0. \quad (2)$$

Appendix A: Max crowding benchmark, \tilde{A}

1. The member trade reallocation that yields the maximum crowding benchmark is

$$\operatorname{argmax}_{\{v_1, v_2, \dots, v_J\}} \min \left(\sum_j v_j^+, \sum_j v_j^- \right) \text{ subject to (1),} \quad (3)$$

where

$$v_j^+ := \max(v_j, 0) \text{ and } v_j^- := \max(-v_j, 0).$$

2. If $\sum_j v_j^+ = \sum_j v_j^-$ then trade reallocation is perfect. No portfolio risk is left unallocated.

Appendix A: Max crowding benchmark, \tilde{A}

1. The trade reallocation is a combinatorial problem that is NP hard.
2. It maps into a one-dimensional bin packing problem (Coffman, Garey, and Johnson, 1996). Can all items be packed into two bins of size $(1/2) \sum_j \sigma_j^2$? If not, how much can be packed into two such bins? The minimum of the two bins can be matched, i.e., buyers buy this amount from sellers.
3. First fit descending (FFD) algorithm solves the offline bin packing problem in $O(J \log J)$ time (brute force requires 3^J).
4. Why FFD instead of alternative approaches?
 - 4.1 Average-case analysis: If item size is drawn from $U[0, 1/2]$ for one-unit bins then Coffman, Garey, and Johnson (1996, p. 39) claim “FFD is typically optimal.”
 - 4.2 Worst-case analysis: If all items are smaller than $1/2$ then FFD does as well its closest contender MFFD (modified first fit descending) (Coffman, Garey, and Johnson, 1996, p. 16-19).

Appendix B: Q&A

1. **Is it reasonable to assume equity returns are normal?** In the implementation the return distribution is assumed to be *conditionally* normal. Time-varying volatility is accounted for by calculating the covariance matrix as an exponentially weighted average of the outer product of historical daily returns.²

²EWMA(0.94) which is the RiskMetrics standard for daily equity returns.

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