Asset Price Dynamics with Limited Attention: Demo Code to Illustrate Estimation of Simplified Statespace Model

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Abstract

This document serves as a `readme.txt` file for a minimal code that estimates a simplified version of the model proposed in Hendershott et al. (2020) (HMSP20). The full code for baseline model is also available but is harder to parse. The minimal code serves as an introduction to the full code and, perhaps more importantly, can serve as a point of departure for researchers interested in estimating their own permutation of the asset-price dynamics model proposed in HMSP20.

1 Preliminary remarks

This python repository demonstrates the code that was used to estimate the statespace model in Hendershott et al. (2020) (HMSP20). It does so by first simulating data from a simplified version of the paper's statespace model and then estimating it using maximum likelihood. The structure of the code is similar to the full code that is also publicly available at, for example, albertjmenkveld.com. The goal of this simple code is two-fold:

1. It serves as an introduction to the full code so that the latter becomes easier to read.
2. It serves as an example of how to estimate an equilibrium model for asset price dynamics using the Kalman filter. It benefits from the well developed statsmodels package in python or, more specifically, the statespace part of that package . Researchers can use it as point of departure for estimation of their own statespace model. See:
   - https://www.statsmodels.org/stable/statespace.html

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- https://www.statsmodels.org/stable/index.html

A couple of further comments clarify the code and how to run it.

1. *Simplified statespace model.* The simplified statespace model used in the code contains only one arrival intensity (instead of many) and removes the "noise terms" in market maker inventory and retail flow. The time unit is one day. Inspired by Hendershott et al. (2020) the model parameters to be estimated are set to:

   (a) Risk-mass of slow institutions (μ_i) is 25.

   (b) Risk-mass of slow retail investors (μ_r) is 5.

   (c) The proportion of market makers in the population of all fast investors (β_M) is 0.01.

   (d) The price-pressure commanded by inefficiently allocated securities (β_w) is 0.10.

   (e) The standard deviation of daily fundamental-value changes (σ_w) is 200 (basis points per day).

   (f) The correlation between fundamental-value changes and changes fast-investors portfolio changes (ρ) is -0.25.

   The pre-set parameters are:

   (a) Arrival intensity (λ) is 0.05 (corresponding to monthly slow investors).

   (b) The discount rate (r) is 0.0002.

   The simplified statespace model therefore becomes:

$$Y_t = \begin{pmatrix} G_{it} & G_{rt} & MMInv_t & RetFlow_t & Return_t \end{pmatrix}^\top \tag{1}$$

   with

$$Y_t = VY_{t-1} + W\varepsilon_t \tag{2}$$

   where

$$V = \begin{pmatrix} e^{-\lambda} & 0 & 0 & 0 & 0 \\ 0 & e^{-\lambda} & 0 & 0 & 0 \\ \beta_M e^{-\lambda} & \beta_M e^{-\lambda} & 0 & 0 & 0 \\ 0 & 1-e^{-\lambda} & 0 & 0 & 0 \\ \beta_w \frac{1-e^{-\lambda}}{r+\lambda} & \beta_w \frac{1-e^{-\lambda}}{r+\lambda} & 0 & 0 & 0 \end{pmatrix}, \tag{3}$$

$$W = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ \beta_M & \beta_M & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 1 \\ \frac{-\beta_w}{r+\lambda} & \frac{-\beta_w}{r+\lambda} & 1 & 0 & 0 \end{pmatrix}, \tag{4}$$

and

$$\mathrm{Cov}\left(\varepsilon_t\right) = \begin{pmatrix} \mu_i^2 A & \rho^2\mu_i\mu_r A & \rho\mu_i\sigma_w B & \mu_i^2 B & \rho^2\mu_i\mu_r B \\ \rho^2\mu_i\mu_r A & \mu_r^2 A & \rho\mu_r\sigma_w B & \rho^2\mu_i\mu_r B & \mu_r^2 B \\ \rho\mu_i\sigma_w B & \rho\mu_r\sigma_w B & \sigma_w^2 & \rho\mu_i\sigma_w & \rho\mu_r\sigma_w \\ \mu_i^2 B & \rho^2\mu_i\mu_r B & \rho\mu_i\sigma_w & \mu_i^2 & \rho^2\mu_i\mu_r \\ \rho^2\mu_i\mu_r B & \mu_r^2 B & \rho\mu_r\sigma_w & \rho^2\mu_i\mu_r & \mu_r^2 \end{pmatrix} \tag{5}$$

where

$$A = \frac{1-e^{-2\lambda}}{2\lambda} \quad \text{and} \quad B = \frac{1-e^{-\lambda}}{\lambda} \tag{6}$$

and

$$\varepsilon_t = \begin{pmatrix} \underset{(1\times 2)}{\varepsilon_{1t}} & \underset{(1\times 1)}{\varepsilon_{2t}} & \underset{(1\times 2)}{\varepsilon_{3t}} \end{pmatrix}, \tag{7}$$

where ε_1 tracks *state changes*, ε_2 tracks fundamental-value (i.e., permanent price changes), and ε_3 tracks flows. Note that ε_1 and ε_3 are positively correlated but fulfill separate roles. ε_1 is needed to compute price pressures whereas ε_2 tracks flows (in this case retail flow is observed).[1]

2. The code structure follows the guidelines of Gentzkow and Shapiro (2014). For example, the code is delivered in four self-explanatory directories: input, code, temp, and output.

3. The code runs on Python 3.8 and possibly older versions.

2 Code output.

Three figures illustrate output of the code based on simulated sample of 1000 days. Figure 1 simply plots the simulated series that enter the estimation procedure. Figure 2 contains a screenshot from the code output with the true model parameters, the starting values, and the parameter estimates as well as their standard errors. Figure 3 contains the signature graph from Hendershott et al. (2020) based on estimation of the model on the simulated series.

References

Gentzkow, Matthew and Jesse M. Shapiro. 2014. "Code and Data for the Social Sciences: A Practitioners Guide." Manuscript, Chicago Booth and NBER.

Hendershott, Terrence, Albert J. Menkveld, Mark Seasholes, and Rémy Praz. 2020. "Asset Price Dynamics with Limited Attention." Manuscript, Vrije Universiteit Amsterdam.

[1] Strictly speaking, in HMSP20 only retail flow is observed and the ε_2 could therefore be replaced by its second element only. Trading by slow institutions still matters as it affects the inefficient allocation (gap) states and this is picked up by the first element of ε_1.

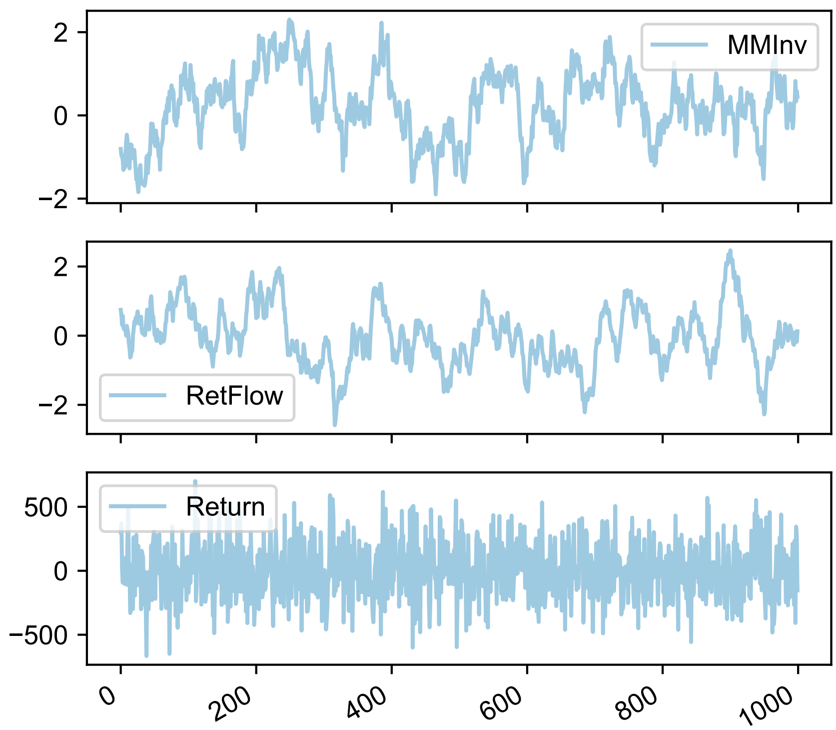

Figure 1: **Simulated series (N=1000).** These plots illustrate the simulated series. This is output from the code.

```
---
The true parameters, starting values, and parameter estimates and their standard errors are:
---
        tr_prmtrs  strtng_vls  prmtr_estmts  stndrd_errrs
m_i      25.00000    14.24365      24.12052       5.03686
m_r       5.00000     4.79325       5.08469       0.11530
bt_M      0.01000     0.01748       0.01044       0.00212
bt_w      0.10000     0.30951       0.10738       0.03697
sgm_w   200.00000   197.86490     197.20429       5.65400
rh       -0.25000    -0.02564      -0.24145       0.04662
---
```

Figure 2: **Screenshot estimation output.** This figure depicts a screenshot from the code that summarizes the estimation output: true model parameters, the starting values that are input to the maximum-likelihood optimization, and the parameter estimates themselves with standard errors.

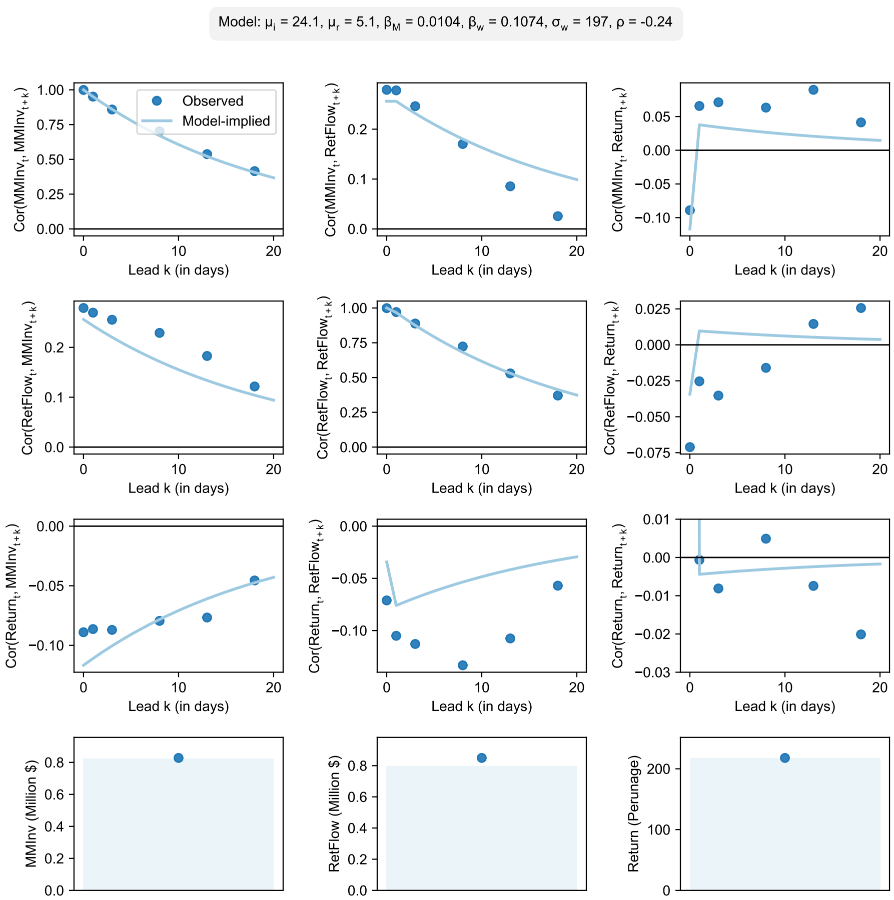

Figure 3: **Observed and estimated model dynamics.** This figure plots the signature graph from Hendershott et al. (2020) based on the code output. Parameter estimates are from Figure 2. The blue lines in the top three rows and blue shaded bars in the bottom row are the model-implied moments. The dark blue dots are the empirical moments.